Exam I - Answers to Review Sheet
MATH 105, Spring 2003

1. Identify the type of notation used to write each of the sets below, and then write the same set in the two remaining notations.

(a) Description
   Set-builder: \( \{ x \mid x \text{ is a color in the rainbow} \} \)
   Roster: \{ Red, Orange, Yello, Green, Blue, Indigo, Violet \}

(b) Set-builder
   Description: The set of all Adventist Colleges in the U.S.
   Roster: \{ Walla Walla, Pacific Union, La Sierra, \ldots \}

(c) Roster
   Description: The set of all prime numbers
   Set-builder: \( \{ x \mid x \text{ is a prime number} \} \)

2. Let \( U \) be the set of all WWC students, the universal set for the following instances.

(a) For example, the set of all students living in Sittner Hall, or the set of all students taking Accounting this quarter.

(b) This is not well defined since the term “friendly” is ambiguous.

(c) Show that every former student missionary has traveled abroad, and every student who has traveled abroad is a former student missionary.

(d) There are \( 2^4 = 16 \) ways you can choose from these people.

(e) There are two correct answers: \( A \) itself, and \( \emptyset \).

3. Consider the weighted voting scheme \( [6 : 4, 3, 2, 1] \).

<table>
<thead>
<tr>
<th>Losing Coalitions</th>
<th>Voter 1</th>
<th>Voter 2</th>
<th>Voter 3</th>
<th>Voter 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \emptyset )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>{1}</td>
<td>-</td>
<td>( \times )</td>
<td>( \times )</td>
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<tr>
<td>{2}</td>
<td>( \times )</td>
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<tr>
<td>{3}</td>
<td>( \times )</td>
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<tr>
<td>{4}</td>
<td>-</td>
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<tr>
<td>{1, 4}</td>
<td>-</td>
<td>( \times )</td>
<td>( \times )</td>
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</tr>
<tr>
<td>{2, 3}</td>
<td>( \times )</td>
<td>-</td>
<td>-</td>
<td>( \times )</td>
</tr>
<tr>
<td>{2, 4}</td>
<td>( \times )</td>
<td>-</td>
<td>( \times )</td>
<td>-</td>
</tr>
<tr>
<td>{3, 4}</td>
<td>( \times )</td>
<td>( \times )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Totals:</strong></td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td><strong>Power:</strong></td>
<td>( 5/12 )</td>
<td>( 3/12 )</td>
<td>( 3/12 )</td>
<td>( 1/12 )</td>
</tr>
</tbody>
</table>

4. Suppose that \( P = \{2, 3, 5, 7, 9, 11\} \), \( E = \{2, 4, 6, 8, 10, 12\} \), \( O = \{1, 3, 5, 7, 9, 11\} \) and \( U = \{1, 2, \ldots 12\} \) is the universal set.

(a) \{1, 2, 3, 5, 7, 9, 11\}  (b) \{3, 5, 7, 9, 11\}  (c) \( O \)
(d) \{4, 6, 8, 10, 12\}  (e) \{2\}  (f) \{1, 2, 3, 5, 7, 9, 11\}
(g) \{1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}  (h) \{2\}  (i) \( P \)
(j) \( E \)  (k) \( E \)  (l) \( \emptyset \)
5. Use Venn Diagrams to determine if the following sets are equal.
   (a) Yes, they are equal.
   (b) No, not equal.
   (c) No, not equal.

6. Suppose that you surveyed 150 students asking them if they attended vespers, church, and chapel during a given week. Of these students, 75 attended vespers, 92 attended church, and 121 attended chapel. 43 attended both vespers and church, 80 attended both church and chapel, and 61 attended both vespers and chapel. 36 attended all three. Construct a Venn Diagram and use it to answer the following questions.

   ![Venn Diagram](image)

   (a) 7 attended vespers and church, but not chapel.
   (b) 10 attended none of the three events.
   (c) 28 attended exactly one event.
   (d) 112 attended at least two of the events.

7. Suppose that \( p \): you vote in the next election, and \( q \): you are an informed elector. State each of the following in English.
   (a) You vote in the next election or are an informed elector.
   (b) You vote in the next election but are not an informed elector.
   (c) You do not either vote in the next election or become an informed elector.
   (d) If you vote in the next election then you are an informed elector.
   (e) You are an informed elector if and only if you vote in the next election.
   (f) If you are not an informed elector, then you vote in the next election.

8. Construct a truth table for each of the statements above.
   Left to the reader

9. Suppose that it is true that \( q \rightarrow p \) using the \( p \) and \( q \) given above. State each of the following in English. Which are equivalent to \( q \rightarrow p \)
   (a) If you vote in the next election, then you are an informed elector. Not equivalent.
   (b) If you are not an informed elector then you do not vote in the next election. Not equivalent.
   (c) If you do not vote in the next election, then you are not an informed elector. Equivalent.

10. Write each argument in symbolic form and determine if it is valid or invalid.
    (a) \( f \lor \neg w, w \), therefore \( f \). Valid
    (b) \( p \rightarrow q \), therefore \( \neg q \rightarrow \neg p \). Valid
    (c) \( (p \lor a) \rightarrow l, \neg l \), therefore \( \neg p \). Valid.