Final Exam Review Sheet  
MATH 105, Spring 2003

This exam will cover sections 3.1-4.5, 5.1,3, and 4, 8.1-9.4, 10.3 and 10.4, and the supplement. You should know general terms and definitions from each of these sections and review the homework, quizzes, and exams given during the quarter.

1. Defining sets using various methods and working with universal sets and subsets.
2. Analysing weighted voting scenarios similar to those found in your text.
3. Combining sets with standard set operations and using Venn diagrams in combining sets.
4. Translating between symbolic logic and English.
5. Working with truth tables, conditional sentences (if... then...) and valid vs. invalid arguments.
6. Describing outcomes, sample spaces, and events for a given experience.
8. Determining probabilities from counting roles, Venn Diagrams, and conditional probabilities.
9. Creating and interpreting histograms and box-plots.
10. Finding mean/standard deviation and 5-number summaries for data sets.
11. Calculating probabilities and proportions using the standard normal probability table.
12. Basic linear equations including slopes, intercepts, graphing, etc...
13. Solving systems of equations including story problems resulting in systems of equations.
14. Graphing regions given by a system of linear inequalities and the finding corner points.
15. Linear programming involving finding the min/max of an objective function on a bounded feasible set.
16. Finding discretely and continuously compounded interest and simple interest.

Below is a list of sample problems. This list is not all-inclusive, but does represent the basic types of problems you may see on the exam.

1. Analyze the weighted voting scheme [7 : 3, 2, 1, 1], finding the power of each voter.

2. Suppose that $A = \{1, 2, a\}$, $B = \{2, 3, b\}$, $C = \{1, a, b, c\}$ and $U = \{1, 2, 3, a, b, c\}$ is the universal set. Find the following sets.
   
   \begin{align*}
   (a) & \quad A \cup C \\
   (b) & \quad A \cap C \\
   (c) & \quad B' \\
   (d) & \quad B - A \\
   (e) & \quad B' - A \\
   (f) & \quad (B - A)' \\
   (g) & \quad (A \cap B)' \\
   (h) & \quad (C \cup A) \cap B \\
   (i) & \quad C \cup (A \cap B)
   \end{align*}

3. Use Venn Diagrams to determine if the following sets are equal.
   
   \begin{align*}
   (a) & \quad A \cup (B \cap C)' \text{ and } (A \cup B') \cap (A \cup C'). \\
   (b) & \quad A \cap B' \text{ and } (A' \cup B)'
   \end{align*}

4. Suppose that $p :$ you run the Boston marathon, and $q :$ you train every day. State each of the following in English and construct truth tables for each.
   
   \begin{align*}
   (a) & \quad p \lor q \\
   (b) & \quad p \land \neg q \\
   (c) & \quad \neg(p \lor q) \\
   (d) & \quad q \rightarrow q \\
   (e) & \quad q \leftrightarrow p \\
   (f) & \quad \neg q \rightarrow p
   \end{align*}

5. Suppose that it is true that $q \rightarrow p$ using the $p$ and $q$ given above. State the Converse, Inverse and Contrapositive of this statement.
6. Of 110 students surveyed, 58 major in science, 40 are upperclassmen, and 52 will attend summer school. 18 upperclassmen major in science, 25 science majors will attend summer school, and 22 upperclassmen will attend summer school. If 15 science majors are upperclassmen and will attend summer school, construct a complete Venn diagram.

7. A government committee of 8: 5 Republicans and 3 Democrats, is to choose a sub-committee of 4. In how many ways can this be done if:
   (a) All are Republicans with a chair, vice-chair, secretary and parliamentarian?
   (b) There are to be 3 Republicans, and 1 Democrat, with no officers?
   (c) There must be more Republicans than Democrats, but are no officers?

8. Referring to the committee problem above, what is the probability that, if the committee is chosen at random with no officers:
   (a) there are exactly two Democrats on the committee?
   (b) there is at least one Democrat and one Republican on the committee?

9. Create a Venn Diagram illustrating the events $E$, $F$, and $G$ are such that $P(E) = .25$, $P(F) = .50$, and $P(G) = .55$. Furthermore, $P(E \cap F) = .10$, $P(E \cap G) = .05$, and $P(F \cap G) = .30$. Finally, $P(E \cap F \cap G) = .05$.

10. An special deck of cards contains the two black aces, one red king, and one red ace. If you draw a card at random, answer the following.
    (a) What is the probability you draw a red card given that you draw a king?
    (b) What is the probability you draw an ace given that you draw a red card?
    (c) What is the probability you draw a black card given that you draw an ace?
    (d) Are any of the pairs of events above independent?

11. A soda company advertising campaign gives prizes away on the inside of bottle caps in 1,000 soda bottles. If 200 bottle caps award $10, 100 award $50 and 10 award $100, what is the expected value of one of these bottle caps?

12. Use the following table of values to perform the tasks listed below.

| 13 | 12 | 11 | 14 | 11 | 10 | 12 | 12 | 13 | 11 |

   (a) Construct a histogram for this data.
   (b) Construct a split-stem plot of this data.
   (c) Give a 5-number summary of this data, and a box-plot.
   (d) Suppose that you sampled the first 3 values from this data. Calculate the mean and standard deviation of these 3 values.

13. Suppose that the height, in inches, of a certain group of individuals follows a normal distribution $\mu = 68$ and $\sigma = 4.5$. If an individual is selected from this group at random, find:
    (a) the probability that the individual is more than 68 inches tall.
    (b) the probability that the individual is between 63.5 and 72.5 inches tall.
    (c) the probability that the individual is between 60 and 68 inches tall.
    (d) the probability that the individual is less than 62 inches tall.
    (e) the probability that the individual is more than 75 inches tall.
14. Find the slope and $y$-intercept of each of the following lines.
   (a) $3y - 6x = 18$
   (b) $y + 2x = 10 - x$
   (c) $3(x + 2) = 5(y - 1)$

15. Graph the lines below by finding the $x$- and $y$-intercepts.
   (a) $x + 3y = 9$
   (b) $2x - y = -4$
   (c) $-x + y = 3$

16. Be able to set-up systems of linear equations and linear programming problems as seen in quizzes, the supplement, and exam 3.

17. Graph the region bounded by the following inequalities.
   (a) $3x + 5y \leq 15$
   $2x + y \leq 6$
   $x \geq 0, y \geq 0$
   (b) $7x + 2y \geq 14$
   $5x + 2y \leq 20$
   $x \geq 0, y \geq 0$
   (c) $2x + 5y \leq 10$
   $-3x + 6y \geq 9$
   $x \geq 0, y \geq 0$

18. If a savings account pays 4.1% interest compounded weekly, how much money will you have if you deposit $4,000 for two years?

19. Calculate the amount of interest paid and monthly payments on a simple interest car loan of $18,000 at 3.9% interest for five years.