Exam I Review Sheet
MATH 105, Spring 2005

This exam will cover sections 6.1-7.3 in your text, omitting section 6.6. You should know general terms and definitions from each of these sections, review the homework and quizzes given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Identifying relations between pairs of sets.
2. Finding the union and intersection of two sets and the complement of a set.
3. Using Venn diagrams to identify sets, count with sets, and work probability problems.
4. Using the counting formula, \( c(A \cup B) = c(A) + c(B) - c(A \cap B) \).
5. Using the multiplication principle of counting.
7. Solving counting problems involving permutations and/or combinations.
8. Finding sample spaces and assigning probabilities to outcomes.
9. Finding probabilities involving equally likely outcomes
10. Finding the probabilities of events and the union of two events.
11. Finding the probability of the complement of an event.
12. Translating between odds and probabilities.
13. Finding the probability of an event using counting techniques.

Below is a list of sample problems. This list is not all-inclusive, but does represent the basic types of problems you may see on the exam.

1. Suppose that \( P = \{2, 3, 5, 7, 9, 11\} \), \( E = \{2, 4, 6, 8, 10, 12\} \), \( O = \{1, 3, 5, 7, 9, 11\} \) and \( U = \{1, 2, \ldots 12\} \) is the universal set. Find the following sets.

   (a) \( P \cup O \)
   (b) \( P \cap O \)
   (c) \( \overline{E} \)
   (d) \( \overline{P \cap E} \)
   (e) \( (O \cup P) \cap E \)
   (f) \( O \cup (P \cap E) \)
   (g) \( U \cap E \)
   (h) \( P \cap E \cap O \)

2. Use Venn Diagrams to determine if the following sets are equal.

   (a) \( A \cup (B \cap C) \) and \( (A \cup B) \cap (A \cup C) \).
   (b) \( A \cap \overline{B} \) and \( \overline{(A \cap B)} \).
   (c) \( A \cup \overline{B} \) and \( \overline{A} \cap B \)

3. Suppose that you surveyed 150 students asking them if they attended vespers, church, and chapel during a given week. Of these students, 75 attended vespers, 92 attended church, and 121 attended chapel. 43 attended both vespers and church, 80 attended both church and chapel, and 61 attended both vespers and chapel. 36 attended all three. Construct a Venn Diagram and use it to answer the following questions.

   (a) How many attended vespers and church, but not chapel?
   (b) How many attended none of the three events?
   (c) How many attended exactly one event?
   (d) How many attended at least two of the events?
4. A government committee of 12: 8 Whigs and 4 Torries, is to choose a sub-committee of 5. In how many ways can this be done if:
   (a) All Whigs are chosen, the first being the chair, second vice-chair, third secretary, and fourth parliamentarian?
   (b) There are to be 3 Whigs, and 2 Torries, with no officers?
   (c) There are to be 3 Whigs, including the chair and vice-chair, and 2 Torries?
   (d) There must be more Whigs than Torries, but are no officers?

5. A family meal at your local takeout restaurant consists of your choice of 3 entrees, 2 salads, 4 drinks, and 1 dessert. In how many ways can you choose the meal if:
   (a) There are 5 of each food item to choose from?
   (b) There are 5 of each food item to choose from, and you do not want BOTH the baked potato entree and the potato salad?
   (c) There are 5 of each food item to choose from, and you decide to eat the entrees in the order you select them?

6. Create a Venn Diagram illustrating each of the probability distributions given below for the given events in a sample space $S$. Be sure to fill in all possible regions on your diagram.
   (a) Events $E$ and $F$ are mutually exclusive, with $Pr[E] = .42$ and $Pr[F] = .16$.
   (b) Events $E$ and $F$ have probabilities $Pr[E] = .65$ and $Pr[F] = .45$ with $Pr[E \cap F] = .20$.
   (c) Events $E$, $F$, and $G$ are such that $Pr[E] = .25$, $Pr[F] = .50$, and $Pr[G] = .55$. Furthermore, $Pr[E \cap F] = .10$, $Pr[E \cap G] = .05$, and $Pr[F \cap G] = .30$. Finally, $Pr[E \cap F \cap G] = .05$.

7. A committee contains 8 women and 4 men. They wish to choose a subcommittee of 4 members. If this subcommittee is chosen at random, what is the probability that:
   (a) the committee is made up of all men?
   (b) there are exactly two men and two women on the committee?
   (c) there is at least one woman on the committee?
   (d) there is at least one man and one woman on the committee?
   (e) Pam, one of the women, makes it onto the committee?