Exam I - Answers to Review Sheet  
MATH 105, Spring 2005

1. Suppose that $P = \{2, 3, 5, 7, 9, 11\}$, $E = \{2, 4, 6, 8, 10, 12\}$, $O = \{1, 3, 5, 7, 9, 11\}$ and $U = \{1, 2, \ldots, 12\}$ is the universal set.

(a) $\{1, 2, 3, 5, 7, 9, 11\}$  
(b) $\{3, 5, 7, 9, 11\}$  
(c) $O$  
(d) $\{1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$  
(e) $\{2\}$  
(f) $P$  
(g) $E$  
(h) $\emptyset$

2. Use Venn Diagrams to determine if the following sets are equal.

(a) Yes, they are equal.  
(b) No, not equal.  
(c) No, not equal.

3. Suppose that you surveyed 150 students asking them if they attended vespers, church, and chapel during a given week. Of these students, 75 attended vespers, 92 attended church, and 121 attended chapel. 43 attended both vespers and church, 80 attended both church and chapel, and 61 attended both vespers and chapel. 36 attended all three. Construct a Venn Diagram and use it to answer the following questions.

(a) 7 attended vespers and church, but not chapel.  
(b) 10 attended none of the three events.  
(c) 28 attended exactly one event.  
(d) 112 attended at least two of the events.

![Venn Diagram]

4. A government committee of 12: 8 Whigs and 4 Torries, is to choose a sub-committee of 5. In how many ways can this be done if:

(a) $P(8, 5) = 6,720$  
(b) $C(8, 3) \times C(4, 2) = 56 \times 6$  
(c) $P(8, 3) \times C(4, 2) = 336 \times 6 = 2,016$  
(d) $C(8, 5) \times C(4, 0) + C(8, 4) \times C(4, 1) + C(8, 3) \times C(4, 2) = 56 \times 1 + 70 \times 4 + 56 \times 6 = 56 + 280 + 336 = 672$

5. A family meal at your local takeout restaurant consists of your choice of 3 entrees, 2 salads, 4 drinks, and 1 dessert. In how many ways can you choose the meal if:

(a) $C(5, 3) \times C(5, 2) \times C(5, 4) \times C(5, 1) = 10 \times 10 \times 5 \times 5 = 2,500$ (assuming no repetitions).  
(b) $2,500 - C(4, 2) \times C(4, 1) \times C(5, 4) \times C(5, 1) = 2,500 - 6 \times 4 \times 5 \times 5 = 2,500 - 600 = 1,900$  
(c) $P(5, 3) \times C(5, 2) \times C(5, 4) \times C(5, 1) = 60 \times 10 \times 5 \times 5 = 15,000$
6. Diagrams are shown below.

7. A committee contains 8 women and 4 men. They wish to choose a subcommittee of 4 members. If this subcommittee is chosen at random, what is the probability that:

(a) All Men: \( \frac{\binom{4}{4}}{\binom{12}{4}} = \frac{1}{495} \approx .002 \)

(b) Two of Each: \( \frac{\binom{8}{2} \times \binom{4}{2}}{\binom{12}{4}} = \frac{168}{495} \approx .339 \)

(c) At least one woman: \( 1 - \frac{\binom{4}{4}}{\binom{12}{4}} \approx 1 - .002 = .998 \)

(d) At least one of each: \( 1 - \frac{\binom{4}{4}}{\binom{12}{4}} - \frac{\binom{8}{4}}{\binom{12}{4}} = 1 - \frac{1}{495} - \frac{70}{495} = \frac{424}{495} \approx .857 \)

(e) Pam on the committee: \( \frac{\binom{11}{3}}{\binom{12}{4}} = \frac{165}{495} \approx .333 \)