Exam III Review Sheet
MATH 105, Spring 2005

This exam will cover sections 1.1, 1.2, and 2.1 through 2.6 in your text. You should know general terms and definitions from each of these sections, review the homework and quizzes given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Graphing lines from given equations.
2. Finding the equation of a line from: a graph, two points, or a point and a slope.
3. Finding the equation of a line parallel or perpendicular to a given line.
4. Solving a system of equations using substitution or elimination.
5. Finding the augmented matrix for a system of equations.
6. Identifying matrices in reduced echelon form.
7. Solving a system of equations using augmented matrices and row operations.
8. Setting up systems of equations in two or three variables from a given story problem.
9. Identifying systems of equations as: consistent+dependent, consistent+independent, or inconsistent.
10. Matrix arithmetic including addition, scalar multiplication, and matrix multiplication.
11. Finding the inverse of a matrix, if it exists.
12. Using the inverse of a matrix to solve a system of equations as a matrix equation.

Below is a list of sample problems. This list is not all-inclusive, but does represent the basic types of problems you may see on the exam.

1. Find the slope, \( x \)-, and \( y \)-intercept of each line below. Then graph each line.
   (a) \( 10x + 15y = 60 \)
   (b) \( 7x - 5y = -35 \)
   (c) \( -6x + 3y = 12 \)

2. Determine the solution type for each of the following systems of linear equations. For those with a unique solution, find that solution using one of the elimination, substitution, augmented matrix, or matrix equation methods. You should use each method at least once.
   \[
   \begin{align*}
   2x + 6y &= 44 \\
   -3x + 5y &= 32 \\
   4x + 6y &= 12 \\
   2x + 3y &= 6 \\
   2x + y + z &= 6 \\
   x - y - z &= -3 \\
   3x + y + 2z &= 7 \\
   8x - 32y &= 15 \\
   4x - 16y &= 21 \\
   5x - \frac{1}{2}y &= -16 \\
   \frac{1}{2}x + 7y &= 13 \\
   z + y - z &= 0 \\
   4x + 4x - 4z &= -1 \\
   2x + y + z &= 2
   \end{align*}
   \]

3. Identify each matrix as being in row echelon form, reduced echelon form, or neither. If a matrix is in neither form, indicate what keeps it from being both row echelon and reduced echelon form.
   \[
   \begin{bmatrix}
   1 & 2 & 0 & 4 \\
   0 & 1 & 1 & 3 \\
   0 & 0 & 0 & 3
   \end{bmatrix}
   \begin{bmatrix}
   1 & 2 & 0 & 4 \\
   0 & 0 & 1 & -1 \\
   0 & 0 & 0 & 3
   \end{bmatrix}
   \begin{bmatrix}
   1 & 0 & 2 & 3 \\
   0 & 0 & 3 & -1
   \end{bmatrix}
   \]
4. Set-up and solve the following story problems involving systems of equations.

(a) A pet food company makes two types of dog food—Hearty Blend and Nature’s Best. The Hearty Blend mixture requires 3 lbs. of rice and 6 lbs. of meat per 10 lb. bag. The Nature’s Best mixture takes 5 lbs. of rice and 4 lbs. of meat per 10 lb. bag. The factory has 135 lbs. of rice and 180 lbs. of meat ready. How many 10 lb. bags of each blend should be made to use up the factory’s supply?

(b) Sally’s Girl Scout troupe is selling cookies for the Christmas season. There are three different kinds of cookies in three different containers: bags hold 1 dozen chocolate chip and 1 dozen oatmeal; gift boxes contain 2 dozen chocolate chip, 1 dozen mint, and 1 dozen oatmeal cookies; and cookie tins hold 3 dozen mint and 2 dozen chocolate chip cookies. Sally’s mother is having a Christmas party and wants 6 dozen oatmeal; 10 dozen mint, and 14 dozen chocolate chip cookies. How can Sally fill her mother’s order?

(c) A store sells almonds for $6 per pound, cashews for $5 per pound, and peanuts for $2 per pound. One week the manager decides to prepare 100 16-ounce packages of nuts by mixing the peanuts, almonds, and cashews. Each package will be sold for $4. The mixture is to produce the same revenue as selling the nuts separately. Prepare a table that shows some of the possible ways the manager can prepare the mixture.

5. Using the matrices \(X\), \(Y\), and \(Z\) defined below to perform the indicated operations, or indicate if an operation is not defined.

\[
X = \begin{bmatrix}
2 & 1 & 3 \\
-1 & 0 & 5
\end{bmatrix}, \quad Y = \begin{bmatrix}
2 & 0 \\
1 & -2
\end{bmatrix}, \quad Z = \begin{bmatrix}
3 & 1 \\
0 & -5 \\
-1 & 2
\end{bmatrix}
\]

(a) \(ZY\) \quad (b) \(YX + 3X\) \quad (c) \(Y - ZY\) \quad (d) \(YXZ\)

6. Which of the following is the values of \(x_2\) in the system of equations represented by \(AX = B\) in which

\[
A^{-1} = \begin{bmatrix}
1 & 3 & -1 \\
2 & 1 & 0 \\
1 & 0 & -2
\end{bmatrix}, \quad X = \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}, \quad B = \begin{bmatrix}
12 \\
9 \\
16
\end{bmatrix}
\]

A. 18 \quad B. 25 \quad C. 33 \quad D. 45 \quad E. None of the above

7. Find the inverse of the matrix \(A\) shown below. Be sure to show all of your work.

\[
A = \begin{bmatrix}
2 & -2 & 0 \\
3 & 0 & -2 \\
0 & 2 & -1
\end{bmatrix}
\]

8. Given that the matrix equation below is correct, what is the value of \(b\)?

\[
\begin{bmatrix}
2 & a & 3 \\
0 & b & -1
\end{bmatrix} \begin{bmatrix}
1 & a \\
0 & 2 \\
-1 & 4
\end{bmatrix} = \begin{bmatrix}
-1 & 16 \\
1 & 2a
\end{bmatrix}
\]