Exam II - Answers to Review Sheet  
MATH 105, Spring 2005

1. A committee contains 8 women and 4 men. They wish to choose a subcommittee of 4 members. If this subcommittee is chosen at random, what is the probability that:
   
   (a) All Men: \( \frac{C(4,4)}{C(12,4)} = \frac{1}{495} \approx .002 \)
   
   (b) Two of Each: \( \frac{C(8,2) \times C(4,2)}{C(12,4)} = \frac{168}{495} \approx .339 \)
   
   (c) At least one woman: \( 1 - \frac{C(4,4)}{C(12,4)} \approx 1 - .002 = .998 \)
   
   (d) At least one of each: \( 1 - \frac{C(4,4)}{C(12,4)} - \frac{C(8,4)}{C(12,4)} = 1 - \frac{1}{495} - \frac{70}{495} = \frac{424}{495} \approx .857 \)
   
   (e) Pam on the committee: \( \frac{C(11,3)}{C(12,4)} = \frac{165}{495} \approx .333 \)

2. An urn contains 12 numbered balls: 8 red and 4 blue. Three of the red balls have even numbers, and 1 of the blue has an even number. The rest of the balls have odd numbers.
   
   (a) \( Pr[red] = \frac{8}{12} = \frac{2}{3} \)
   
   (b) \( Pr[red|even] = \frac{3}{12} = \frac{1}{4} \)
   
   (c) \( Pr[odd] = \frac{8}{12} = \frac{2}{3} \)
   
   (d) \( Pr[odd|blue] = \frac{3}{12} = \frac{1}{4} \)
   
   (e) No - both probabilities change

3. In the experiment above, suppose that you pay $1.00 to draw a ball. An odd numbered red ball gets you nothing, an even numbered red pays $1.00 back, an odd numbered blue gets $5.00, and the even numbered blue gets you $10.00. What is the expected value?

<table>
<thead>
<tr>
<th>Value</th>
<th>Probability</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>5/12</td>
<td>-5/12</td>
</tr>
<tr>
<td>0</td>
<td>3/12</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>3/12</td>
<td>12/12</td>
</tr>
<tr>
<td>9</td>
<td>1/12</td>
<td>9/12</td>
</tr>
<tr>
<td>Total:</td>
<td>16/12</td>
<td>1.33</td>
</tr>
</tbody>
</table>

4. A widget factory creates two types of widgets: type A and type B. Type A widgets are defective 5% of the time, and type B widgets are defective 10% of the time. If the factory produces 20 type A and 60 type B widgets in a given day, and a randomly selected widget is defective, what is the probability that it is type A? If a randomly selected widget is not defective, what is the probability it is type B?

   \[ \begin{align*}
   &A & &N & &D & &\text{.0125} \\
   &20/80 & & & & & & \text{.95} \\
   & & & & & & & \text{.95} \\
   & & & & & & & \text{.2375} \\
   &B & & & & & & \text{.075} \\
   &60/80 & & & & & & \text{.10} \\
   & & & & & & & \text{.90} \\
   & & & & & & & \text{.676} \\
   \end{align*} \]

   (a) \( Pr[A|D] = \frac{.0125}{.0125 + .075} \approx .4129 \)
   
   (b) \( Pr[B|N] = \frac{.676}{.2375 + .676} \approx .7400 \)
5. A student takes a multiple choice test made up of 10 questions. Each question has five possible answers, one of which is correct.

(a) \( Pr[6 \text{ correct}] = C(10, 6) \left( \frac{1}{5} \right)^6 \left( \frac{4}{5} \right)^4 \approx 0.006 \)

(b) \( Pr[\text{at least 8}] = C(10, 8) \left( \frac{1}{5} \right)^8 \left( \frac{4}{5} \right)^2 + C(10, 9) \left( \frac{1}{5} \right)^9 \left( \frac{4}{5} \right) + C(10, 10) \left( \frac{1}{5} \right)^{10} \left( \frac{4}{5} \right)^0 \approx 0.000078 \)

(c) Expected Score = \( np = 10 \left( \frac{1}{5} \right) = 2 \)

(d) Normal Distribution with \( \mu = np = 50 \left( \frac{1}{5} \right) = 10 \) and \( \sigma = \sqrt{npq} = \sqrt{30 \left( \frac{1}{5} \right) \left( \frac{4}{5} \right)} \approx 2.83 \), the score of 15 gives a \( Z \)-score \( Z = \frac{x - \mu}{\sigma} = \frac{15 - 10}{2.83} \approx 1.77 \) and from the table, the area above 1.77 is .5 - .4616 = .0384. So, the probability of scoring 15 or more is only .0384, or 3.84%.

6. Answers follow

(b) Median: 12, Mode: Trimodal with 11, 12, and 18.

(c) Mean: \( \bar{X} = \frac{15+11+18+6}{4} = 12.5 \), StDev: \( S = \sqrt{\frac{(15-12.5)^2+(11-12.5)^2+(18-12.5)^2+(6-12.5)^2}{4-1}} \approx 5.2 \)

(d) 6 is an outlier, new Mean: \( \frac{15+11+18}{3} \approx 14.67 \) and StDev \( \sqrt{\frac{(15-12.5)^2+(11-12.5)^2+(18-12.5)^2}{3-1}} \approx 4.40 \)

7. Suppose that the weights, in pounds, of a certain group of individuals follows a normal distribution \( \mu = 175 \) and \( \sigma = 12.5 \). If an individual is selected from this group at random, find:

(a) .5

(b) .68 (use empirical rule)

(c) \( Z = \frac{155-175}{12.5} = -1.6 \) so area is 0.4425.

(d) \( Z = -1.6 \) again, so area is 0.5 - 0.4425 = 0.0575.

(e) \( Z_1 = \frac{190-175}{12.5} = 1.2 \) and \( Z_2 = \frac{210-175}{12.5} = 2.8 \) so area is 0.4974 - 0.3849 = 0.1125