MATH 105: Finite Mathematics
1-1: Rectangular Coordinates, Lines

Prof. Jonathan Duncan
Walla Walla College
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Outline

1. Rectangular Coordinate System
2. Graphing Lines
3. The Equation of a Line
4. Conclusion
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The Cartesian Coordinate System, also called the rectangular coordinate system, is shown below.

![Diagram of the Cartesian Coordinate System](image)
Points on the coordinate system are located using an $x$-coordinate and $y$-coordinate. They are grouped together into a pair of numbers, $(x, y)$.

Plot each of the following points.

$P = (-3, 5)$

$R = (2, 0)$

$S = (-1, -2)$
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Graphing a Set of Points

We are particularly interested in graphing lines. A line is just a particular set of points.

The Graph of a Line

The graph of a line is the graph obtained by plotting all points in the set

\[ \{(x, y) \mid Ax + By = C\} \]

where \(A\), \(B\), and \(C\) are real numbers.

The General Equation of a Line

The equation \(Ax + By = C\) is called the general equation of a line.

A Point on a Line

A point \((x_1, y_1)\) is on the line \(Ax + By = C\) if \(Ax_1 + By_1 = C\) is a true statement.
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Graphing a Line

Since a line is nothing more than a set of points, we can graph it by determining a few of those points and then connecting them.

Graph the line represented by $2x - y = 4$
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<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>
Graphing a line can be made a lot easier by using the following two points:

**x-intercept**

The x-intercept of a line is the point at which the line crosses the x-axis, where $y = 0$.

**y-intercept**

The y-intercept of a line is the point at which the line crosses the y-axis, where $x = 0$. 
Intercepts of a Line

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Finding the $x$ and $y$ intercepts is relatively easy and usually produces the two points needed to graph a line.
Use $x$- and $y$-intercepts to graph the line $3x + 5y = 15$.

- $y$-intercept: $3$
- $x$-intercept: $5$
Graphing Lines using Intercepts

Use $x$- and $y$-intercepts to graph the line $3x + 5y = 15$.

$y$-intercept: 3  
$x$-intercept: 5
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Graphing

Use $x$- and $y$-intercepts to graph the line $7x - 4y = 28$.

$y$-intercept: -7
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The Slope Equation

The angle of a line is referred to as the slope of the line. It can be found by dividing the change in $y$ by the change in $x$.

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
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$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
The Slope Equation

The angle of a line is referred to as the slope of the line. It can be found by dividing the change in $y$ by the change in $x$.

![Diagram showing the slope of a line](image)

**Slope**

The slope of a line containing points $(x_1, y_1)$ and $(x_2, y_2)$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
Finding Slopes

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Find the slope of the following line.

\[ m = \frac{3 - 0}{0 - 5} = -\frac{3}{5} \]
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The Slope-Intercept Equation of a Line

Deriving the Slope-Intercept Form

\[ Ax + By = C \]
\[ By = -Ax + C \]
\[ y = -\frac{A}{B}x + \frac{C}{B} \]

Note that if we plug in 0 for \( x \), then \( y = \frac{C}{B} \), so \( (0, \frac{C}{B}) \) is a point on the line, the \( y \)-intercept.

Slope-Intercept Equation of a Line

An equation of a line with slope \( m \) and \( y \)-intercept \((0, b)\) is

\[ y = mx + b \]
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\[ y = mx + b \]
Examples of Finding Slopes

By putting an equation into slope-intercept form, it is possible to read the slope of the line directly from the equation.

Finding Slopes

Find the slope of each equation by writing the equation in slope-intercept form.

1. $3x + 5y = 15$
2. $7x - 4y = 28$
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Find the slope of each equation by writing the equation in slope-intercept form.

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y = -\frac{3}{5}x + 3 \Rightarrow m = -\frac{3}{5}
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Find the slope of each equation by writing the equation in slope-intercept form.

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2. $7x - 4y = 28$
   
   $y = \frac{7}{4}x - 7 \Rightarrow m = \frac{7}{4}$
Some Special Slopes

Horizontal and vertical lines have special slopes. To see this, recall that the slope of a line $Ax + By = C$ is given by $-\frac{A}{B}$.

**Slope of a Horizontal Line**
A horizontal line with equation $y = a$ has slope $m = 0$.

**Slope of a Vertical Line**
A vertical line with equation $x = b$ has an undefined slope.
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A horizontal line with equation $y = a$ has slope $m = 0$.

**Slope of a Vertical Line**

A vertical line with equation $x = b$ has an undefined slope.
We can also use a given slope and point to write the equation for a line.

Point-Slope Equation of a Line

An equation of a nonvertical line with slope $m$ that contains the point $(x_1, y_1)$ is:

$$y - y_1 = m(x - x_1)$$
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Using the formula for slope between a given point $(x_1, y_1)$ and an arbitrary point $(x, y)$ together with a given slope $m$ gives the equation above.
Finding the Equation of a Line

Now that we have seen three different forms of the equation for a line, we can use whichever one is most appropriate.

**Finding Equations**

Find the general equation for each line described below.

1. Line with slope $m = -\frac{2}{3}$ through $(2, 4)$
2. Line through points $(2, 5)$ and $(1, 2)$
3. Line with slope $\frac{1}{3}$ with $y$-intercept $-2$
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   \[
   y - 4 = -\frac{2}{3}(x - 2) \Rightarrow 2x + 3y = 6
   \]

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   \[ m = \frac{5 - 2}{2 - 1} = \frac{3}{1} \Rightarrow y - 2 = 3(x - 1) \Rightarrow 3x - y = 1 \]

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3. Line with slope \( \frac{1}{5} \) with \(y\)-intercept \(-2\).

\[
y = \frac{1}{5}x - 2 \Rightarrow x - 5y = 10
\]
Things to Remember from Section 1-1

1. Graphing Lines: Find the Intercepts!

2. Equations of a Line:
   - General Equation: $Ax + By = C$
   - Slope-Intercept: $y = mx + b$
   - Point-Slope: $y - y_1 = m(x - x_1)$
## Important Concepts

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- Prepare for a quiz on section 1-1.
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