Reduced Row Echelon Form
Systems with More Variables than Equations
Conclusion

MATH 105: Finite Mathematics
2-3: Systems of $m$ Equations with $n$ Variables

Prof. Jonathan Duncan

Walla Walla College

Winter Quarter, 2006
Outline

1. Reduced Row Echelon Form
2. Systems with More Variables than Equations
3. Conclusion
Reduced Row Echelon Form

We saw in the last section that putting augmented matrices into row echelon form makes it easier to solve the associated equations. In this section we will use an even more specific form of an augmented matrix.

Reduced Row Echelon Form of a Matrix

An augmented matrix is in reduced row echelon form if:

1. The first nonzero entry in each row is a 1 and has 0s above and below it.
2. The leftmost 1 of any row is to the right of the leftmost 1 in the row above.
3. Any rows that contain all 0s to the left of the vertical bar appear at the bottom.
We saw in the last section that putting augmented matrices into row echelon form makes it easier to solve the associated equations. In this section we will use an even more specific form of an augmented matrix.

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## Identifying Reduced Row Echelon Form

### Example

Which of the following matrices are in reduced row echelon form?

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

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<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Identifying Reduced Row Echelon Form

Example

Which of the following matrices are in reduced row echelon form?

\[
\begin{bmatrix}
1 & 0 & 2 \\
0 & 1 & 3 \\
\end{bmatrix}
\quad
\begin{bmatrix}
1 & 0 & -3 & | & 0 \\
0 & 1 & 2 & & 1 \\
0 & 0 & 0 & & 0 \\
\end{bmatrix}
\quad
\begin{bmatrix}
1 & 0 & 0 & | & 0 \\
0 & 0 & 1 & & 1 \\
0 & 1 & 0 & & 0 \\
\end{bmatrix}
\quad
\begin{bmatrix}
1 & 0 & 2 & 4 \\
0 & 1 & 3 & 3 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]
Identifying Reduced Row Echelon Form

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1 & 0 & 2 \\
0 & 1 & 3 \\
1 & 0 & 2 \\
0 & 1 & 1 \\
0 & 0 & 0 \\
\end{bmatrix}
\quad
\begin{bmatrix}
1 & 0 & -3 & 0 \\
0 & 1 & 2 & 1 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 \\
\end{bmatrix}
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1 & 0 & 2 \\
0 & 1 & 1 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & -3 & 0 \\
0 & 1 & 2 & 1 \\
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 \\
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0 & 1 & 2 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & 1 & 0 & 0
\end{bmatrix}
\]
Solving with Reduced Row Echelon Form

When reduced row echelon form is used, the solution can be read right off of the augmented matrix.

Example
Tom and Sally open a fruit drink stand. They have 45 lemons and 30 oranges to use in making tart and sweet drink. Two lemons and one orange are needed to make 10 glasses of tart drink. One lemon and 3 oranges are needed to make 10 glasses of sweet drink. How many glasses of each should be made to use up all the fruit?
When reduced row echelon form is used, the solution can be read right off of the augmented matrix.

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<table>
<thead>
<tr>
<th>Oranges</th>
<th>Tart (x)</th>
<th>Sweet (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x</td>
<td>3y</td>
</tr>
<tr>
<td></td>
<td>2x</td>
<td>y</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lemons</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
</tr>
</tbody>
</table>
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Tom and Sally open a fruit drink stand. They have 45 lemons and 30 oranges to use in making tart and sweet drink. Two lemons and one orange are needed to make 10 glasses of tart drink. One lemon and 3 oranges are needed to make 10 glasses of sweet drink. How many glasses of each should be made to use up all the fruit?

\[
\begin{array}{c|cc}
\text{Oranges} & \text{Tart (x)} & \text{Sweet (y)} \\
\text{Lemons} & x & 3y \\
& 2x & y \\
\end{array}
\]

\[
\begin{bmatrix}
30 & 1 & 0 & 21 \\
0 & 1 & 3 \\
\end{bmatrix}
\]
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<th>30 1 0 21</th>
</tr>
</thead>
<tbody>
<tr>
<td>45 0 1 3</td>
</tr>
</tbody>
</table>

They should make 21 batches of tart and 3 batches of sweet drink.
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Solving a System with More Variables than Equations

Example

Solve the following system of equations using augmented matrices and reduced row echelon form.

\[
\begin{align*}
x_1 + 2x_2 + 3x_3 - x_4 &= 0 \\
3x_1 - x_4 &= 4 \\
x_2 - x_3 - x_4 &= 2
\end{align*}
\]
Solving a System with More Variables than Equations

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  x_1 + 2x_2 + 3x_3 - x_4 &= 0 \\
  3x_1 - x_4 &= 4 \\
  x_2 - x_3 - x_4 &= 2
\end{align*} \]

\[ \begin{align*}
  x_1 &= \frac{1}{3}x_4 - \frac{28}{3} \\
  x_2 &= \frac{11}{15}x_4 + \frac{46}{15}
\end{align*} \]
Example

A doctor’s prescription calls for the creation of a pill which contains 12 units of vitamine $B_{12}$ and 12 units of vitamin $E$. Your pharmacy stocks three powders which can be used to make the pill. One contains 20% vitamin $B_{12}$ and 30% vitamin $E$. A second contains 40% vitamin $B_{12}$ and 20% vitamin $E$. The third has 30% vitamin $B_{12}$ and 40% vitamin $E$. How could these powders be mixed to make the required pill?
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<table>
<thead>
<tr>
<th>Vitamin</th>
<th>$B_{12}$</th>
<th>$E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>.20$x$</td>
<td>.30z</td>
</tr>
<tr>
<td>$y$</td>
<td>.40$y$</td>
<td></td>
</tr>
<tr>
<td>$z$</td>
<td>.20$z$</td>
<td>.40z</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>.20x</td>
<td>.40y</td>
<td>.30z</td>
<td>12</td>
</tr>
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Important Concepts

Things to Remember from Section 2-3

1. Reduced Row Echelon Form
2. Solving Systems of Equations with more Variables than Equations
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Next Time...

In section 2-4 we will start to look at matrices as mathematical objects which we can combine through addition, multiplication, and so on.

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