The Matrix Form of a System of Equations

Matrix Multiplication

The Identity Matrix

Conclusion

MATH 105: Finite Mathematics
2-5: Matrix Multiplication

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Winter Quarter, 2006
Outline

1. The Matrix Form of a System of Equations
2. Matrix Multiplication
3. The Identity Matrix
4. Conclusion
Recall that we started working with matrices to make it easier to solve a system of equations.

Matrix Equations

Write the following system of equations as a matrix equation.
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Write the following system of equations as a matrix equation.

\[ \begin{align*}
2x + 3y &= 7 \\
3x - 4y &= 2
\end{align*} \]
Systems of Equations

Recall that we started working with matrices to make it easier to solve a system of equations.

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\begin{bmatrix}
2 & 3 \\
3 & -4
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
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\]

If we want these two expressions to mean the same thing, then the multiplication of the two matrices must yield:

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2x + 3y \\
3x - 4y
\end{bmatrix}
= 
\begin{bmatrix}
7 \\
2
\end{bmatrix}
\]
Multiplying Columns and Rows

Example

Expanding the rule above, multiply the $1 \times 3$ row vector by the $3 \times 1$ column vector as shown below.

$$\begin{bmatrix} 2 & 4 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 5 \end{bmatrix}$$
Example

Expanding the rule above, multiply the $1 \times 3$ row vector by the $3 \times 1$ column vector as shown below.

$$\begin{bmatrix} 2 & 4 & 0 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ 5 \end{bmatrix}$$

$$= 2(-3) + 4(1) + 0(5) = -2$$
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If we know how to multiply a row vector by a column vector, we can use that to define matrix multiplication in general.

Matrix Multiplication

If \( A \) is an \( m \times n \) matrix and \( B \) is an \( n \times k \) matrix, then the product \( AB \) is defined to be the \( m \times k \) matrix whose entry in the \( i \)th row, \( j \)th column is the sum of the products of the \( i \)th row of \( A \) and \( j \)th column of \( B \).

Things to Notice:

- The matrices must have matching “inner” dimensions.
- The new matrix has the “outer” dimensions of the two matrices.
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1. The matrices must have matching “inner” dimensions.
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Examples of Matrix Multiplication

Multiply

Find the product of the matrices below, if possible.

1. \[
\begin{bmatrix}
1 & -2 & 3 \\
4 & 0 & 6 \\
\end{bmatrix}
\begin{bmatrix}
-1 & 2 & 1 \\
1 & 3 & 0 \\
0 & 4 & -2 \\
\end{bmatrix}
\]

2. \[
\begin{bmatrix}
2 & 3 \\
4 & 1 \\
\end{bmatrix}
\begin{bmatrix}
2 & 5 \\
7 & 3 \\
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\[
\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 7 & 3 \\ 1 & 4 \end{bmatrix}
\]
Properties of Matrix Multiplication

Matrix multiplication does not have all the same properties as multiplication of numbers.

Matrix Multiplication is Not Commutative

Use the matrices $A$ and $B$ given below to show that matrix multiplication is not commutative.

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 1 \\ 1 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} -5 & 4 \\ 4 & 8 \end{bmatrix}$$

$$BA = \begin{bmatrix} -5 & 4 \\ 4 & 9 \end{bmatrix}$$
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While matrix multiplication may not be commutative, there are some properties from the multiplication of real numbers which do still hold.

**Properties that DO Work**

If $A$, $B$, and $C$ are matrices of the appropriate dimension then,

- $A(BC) = (AB)C$ (Associative Property)
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Multiplying by 1

When multiplying real numbers, the number 1 is special because for any real number \( a \), \( 1 \cdot a = a \cdot 1 = a \). Because of this, 1 is called the **identity** for multiplication.

**Identity Matrix**

For any positive integer \( n \), the **identity matrix**, \( I_n \), is an \( n \times n \) square matrix with 1s on the top-left to bottom-right diagonal and 0s elsewhere.

**Checking the Identity**

Show that \( I_3 \) works as an identity matrix for the matrix

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Important Concepts

Things to Remember from Section 2-5

1. Matrix Multiplication and Dimensions
2. Multiplying Matrices
3. Matrix Multiplication is not Commutative.
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