MATH 105: Finite Mathematics
2-6: The Inverse of a Matrix

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Recall that last time we saw that a system of equations can be represented as a matrix equation as shown below.

Example

Write the following system of equations in matrix form.
Recall that last time we saw that a system of equations can be represented as a matrix equation as shown below.

**Example**

Write the following system of equations in matrix form.

\[
\begin{align*}
2x & + 3y = 7 \\
3x & - 4y = 2
\end{align*}
\]
Recall that last time we saw that a system of equations can be represented as a matrix equation as shown below.

Example
Write the following system of equations in matrix form.

\[
\begin{align*}
2x + 3y &= 7 \\
3x - 4y &= 2
\end{align*}
\]

\[
\begin{bmatrix}
2 & 3 \\
3 & -4
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
7 \\
2
\end{bmatrix}
\]
Recall that last time we saw that a system of equations can be represented as a matrix equation as shown below.

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Write the following system of equations in matrix form.

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\end{bmatrix}
\]

\[AX = B\]
Recall that last time we saw that a system of equations can be represented as a matrix equation as shown below.

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Write the following system of equations in matrix form.

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x \\
y
\end{bmatrix}
= 
\begin{bmatrix}
7 \\
2
\end{bmatrix}
= AX = B
\]

If we wish to use the matrix equation on the right to solve a system of equations, then we need to review how we solve basic equations involving numbers.
The most basic algebra equation, $ax = b$, is solved using the multiplicative inverse of $a$.

**Example**

Solve the equation $3x = 6$ for $x$.

| Step 1: Multiply by $\frac{1}{3}$ | $\frac{1}{3} \cdot (3x) = \frac{1}{3} \cdot (6)$ |
| Step 2: Simplify the Right | $1 \cdot x = 2$ |
| Step 3: Simplify the Left | $x = 2$ |
| Step 4: Solution | $x = 2$ |
Solving a Simple Equation

The most basic algebra equation, $ax = b$, is solved using the multiplicative inverse of $a$.

Example

Solve the equation $3x = 6$ for $x$.

Step 1: Multiply by $\frac{1}{3}$

$$\frac{1}{3} \cdot (3x) = \frac{1}{3} \cdot (6)$$

Step 2: Simplify the Right

$$\left(\frac{1}{3} \cdot 3\right) x = 2$$

Step 3: Simplify the Left

$$1 \cdot x = 2$$

Step 4: Solution

$$x = 2$$
Solving a Simple Equation

The most basic algebra equation, $ax = b$, is solved using the multiplicative inverse of $a$.

**Example**

Solve the equation $3x = 6$ for $x$.

<table>
<thead>
<tr>
<th>Step 1: Multiply by $\frac{1}{3}$</th>
<th>$\frac{1}{3} \cdot (3x) = \frac{1}{3} \cdot (6)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 2: Simplify the Right</td>
<td>$(\frac{1}{3} \cdot 3) x = 2$</td>
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The most basic algebra equation, \( ax = b \), is solved using the multiplicative inverse of \( a \).

Example

Solve the equation \( 3x = 6 \) for \( x \).

Step 1: Multiply by \( \frac{1}{3} \)

\[
\frac{1}{3} \cdot (3x) = \frac{1}{3} \cdot (6)
\]

Step 2: Simplify the Right

\[
(\frac{1}{3} \cdot 3)x = 2
\]

Step 3: Simplify the Left

\[
1 \cdot x = 2
\]

Step 4: Solution

\[
x = 2
\]
The most basic algebra equation, \( ax = b \), is solved using the multiplicative inverse of \( a \).

Example

Solve the equation \( 3x = 6 \) for \( x \).

Step 1: Multiply by \( \frac{1}{3} \)

\[ \frac{1}{3} \cdot (3x) = \frac{1}{3} \cdot (6) \]

Step 2: Simplify the Right

\[ \left( \frac{1}{3} \cdot 3 \right) x = 2 \]

Step 3: Simplify the Left

\[ 1 \cdot x = 2 \]

Step 4: Solution

\[ x = 2 \]
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The most basic algebra equation, $ax = b$, is solved using the multiplicative inverse of $a$.

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Step 2: Simplify the Right

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Step 3: Simplify the Left

\[
1 \cdot x = 2
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Step 4: Solution

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x = 2
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Solving a Simple Equation

The most basic algebra equation, \( ax = b \), is solved using the multiplicative inverse of \( a \).

Example

Solve the equation \( 3x = 6 \) for \( x \).

Step 1: Multiply by \( \frac{1}{3} \)
\[
\frac{1}{3} \cdot (3x) = \frac{1}{3} \cdot (6)
\]

Step 2: Simplify the Right
\[
(\frac{1}{3} \cdot 3) \cdot x = 2
\]

Step 3: Simplify the Left
\[
1 \cdot x = 2
\]

Step 4: Solution
\[
x = 2
\]

This solution process worked because \( \frac{1}{3} \) is the inverse of 3, so that \( \frac{1}{3} \cdot 3 = 1 \), the identity for multiplication.
Matrix Inverse

To solve the matrix equation $AX = B$ we need to find a matrix which we can multiply by $A$ to get the identity $I_n$.

**Matrix Inverse**

Let $A$ be an $n \times n$ matrix. Then a matrix $A^{-1}$ is the inverse of $A$ if $AA^{-1} = A^{-1}A = I_n$.

**Caution:**

Just as with numbers, not every matrix will have an inverse!
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Caution:

Just as with numbers, not every matrix will have an inverse!
Verifying Matrix Inverses

Example

Show that \[
\begin{bmatrix}
-1 \\ 3
\end{bmatrix}
\begin{bmatrix}
-2 \\ 4
\end{bmatrix}
\text{ and }
\begin{bmatrix}
2 \\ -3/2
\end{bmatrix}
\begin{bmatrix}
1 \\ -1/2
\end{bmatrix}
\] are inverses.

1. \[
\begin{bmatrix}
-1 & -2 \\ 3 & 4
\end{bmatrix}
\begin{bmatrix}
2/3 & 1/2 \\ -3/2 & -1/2
\end{bmatrix}
= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
= I_2
\]

2. \[
\begin{bmatrix}
2 \\ -3/2
\end{bmatrix}
\begin{bmatrix}
-1 \\ 3
\end{bmatrix}
\begin{bmatrix}
-2 \\ 4
\end{bmatrix}
= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
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2 & 1 \\
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Show that \[
\begin{bmatrix}
-1 & -2 \\
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1. \[
\begin{bmatrix}
-1 & -2 \\
3 & 4
\end{bmatrix}
\begin{bmatrix}
2 & 1 \\
-\frac{3}{2} & -\frac{1}{2}
\end{bmatrix}
= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2
\]

2. \[
\begin{bmatrix}
2 & 1 \\
-\frac{3}{2} & -\frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
-1 & -2 \\
3 & 4
\end{bmatrix}
= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2
\]
Verifying Matrix Inverses

Example

Show that \( \begin{bmatrix} -1 & -2 \\ 3 & 4 \end{bmatrix} \) and \( \begin{bmatrix} 2 & 1 \\ -\frac{3}{2} & -\frac{1}{2} \end{bmatrix} \) are inverses.

\[
\begin{bmatrix} -1 & -2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -\frac{3}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2
\]

\[
\begin{bmatrix} 2 & 1 \\ -\frac{3}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2
\]

While it is relatively easy to verify that matrices are inverses, we really need to be able to find the inverse of a given matrix.
Finding a Matrix Inverse

To find the inverse of a matrix \( A \) we will use the fact that \( AA^{-1} = I_n \).

**Find \( A^{-1} \)**

Let \( A = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} \) and find \( A^{-1} = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} \).

\[
AA^{-1} = I_2 \Rightarrow \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

\[
\begin{bmatrix} 3x_1 + 2x_3 & 3x_2 + 2x_4 \\ -x_1 + 4x_3 & -x_2 + 4x_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

This gives two systems of equations:

\[
\begin{align*}
3x_1 + 2x_3 &= 1 \\
-x_1 + 4x_3 &= 0
\end{align*}
\]

\[
\begin{align*}
3x_2 + 2x_4 &= 0 \\
-x_2 + 4x_4 &= 1
\end{align*}
\]


Finding a Matrix Inverse

To find the inverse of a matrix $A$ we will use the fact that $AA^{-1} = I_n$.

Let $A = \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$ and find $A^{-1} = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$.

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$$\begin{bmatrix} 3x_1 + 2x_3 & 3x_2 + 2x_4 \\ -x_1 + 4x_3 & -x_2 + 4x_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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$$\begin{cases} 3x_1 + 2x_3 = 1 \\ -x_1 + 4x_3 = 0 \end{cases}$$

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To find the inverse of a matrix $A$ we will use the fact that $AA^{-1} = I_n$.

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$$\begin{bmatrix} 3x_1 + 2x_3 & 3x_2 + 2x_4 \\ -x_1 + 4x_3 & -x_2 + 4x_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

This gives two systems of equations:

$$\begin{cases} 3x_1 + 2x_3 = 1 \\ -x_1 + 4x_3 = 0 \end{cases} \quad \begin{cases} 3x_2 + 2x_4 = 0 \\ -x_2 + 4x_4 = 1 \end{cases}$$
Finding the Matrix Inverse, Cont.

Solve the systems of equations:

\[
\begin{align*}
3x_1 + 2x_3 &= 1 \\
-x_1 + 4x_3 &= 0
\end{align*}
\]

\[
\begin{align*}
3x_2 + 2x_4 &= 0 \\
-x_2 + 4x_4 &= 1
\end{align*}
\]

\[
\begin{bmatrix}
3 & 2 & | & 1 & 0 \\
-1 & 4 & | & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & | & \frac{4}{14} & -\frac{2}{14} \\
0 & 1 & | & \frac{1}{14} & \frac{3}{14}
\end{bmatrix}
\]

\[
A^{-1} = \begin{bmatrix}
\frac{4}{14} & -\frac{2}{14} \\
\frac{1}{14} & \frac{3}{14}
\end{bmatrix}
\]
Finding the Matrix Inverse, Cont.

Solve the systems of equations:

\[
\begin{align*}
3x_1 + 2x_3 &= 1 \\
-x_1 + 4x_3 &= 0
\end{align*}
\]

\[
\begin{align*}
3x_2 + 2x_4 &= 0 \\
-x_2 + 4x_4 &= 1
\end{align*}
\]

\[
\begin{bmatrix}
3 & 2 & | & 1 & 0 \\
-1 & 4 & | & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & | & \frac{4}{14} & -\frac{2}{14} \\
0 & 1 & | & \frac{1}{14} & \frac{3}{14}
\end{bmatrix}
\]

\[
A^{-1} = \begin{bmatrix}
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\end{bmatrix}
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Finding the Matrix Inverse, Cont...

Finding a Matrix Inverse, Continued

Solve the systems of equations:

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\begin{align*}
3x_1 + 2x_3 &= 1 \\
-x_1 + 4x_3 &= 0
\end{align*}
\]

\[
\begin{align*}
3x_2 + 2x_4 &= 0 \\
x_2 + 4x_4 &= 1
\end{align*}
\]

\[
\begin{bmatrix}
3 & 2 & | & 1 & 0 \\
-1 & 4 & | & 0 & 1
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & | & \frac{4}{14} & -\frac{2}{14} \\
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\]

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A^{-1} = \begin{bmatrix}
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\end{bmatrix}
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Finding the Matrix Inverse, Cont.

Find a Matrix Inverse, Continued

Solve the systems of equations:

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3x_1 + 2x_3 &= 1 \\
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\end{align*}
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\[
\begin{align*}
3x_2 + 2x_4 &= 0 \\
-x_2 + 4x_4 &= 1
\end{align*}
\]

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\begin{bmatrix}
3 & 2 & | & 1 & 0 \\
-1 & 4 & | & 0 & 1
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1 & 0 & | & \frac{4}{14} & -\frac{2}{14} \\
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\]

\[
A^{-1} = \begin{bmatrix}
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\end{bmatrix}
\]
General Rule for Finding An Inverse

Applying the lessons of the previous example yields a general procedure for finding the inverse of a matrix.

**Finding a Matrix Inverse**

To find the inverse of a $n \times n$ matrix $A$, form the augmented matrix $[A \mid I_n]$ and use row reduction to transform it into $[I_n \mid A^{-1}]$.

**Caution:**

It may not be possible to get $I_n$ on the left side of the matrix. If it is not possible, then the matrix $A$ has no inverse.
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Finding the Inverse of a $3 \times 3$ Matrix

Example

Find the inverse of the following matrix, if it exists.

$$
\begin{bmatrix}
1 & 1 & 1 \\
3 & -4 & 2 \\
0 & 0 & 0 \\
\end{bmatrix}
$$
Finding the Inverse of a 3 × 3 Matrix

Example

Find the inverse of the following matrix, if it exists.

\[
\begin{bmatrix}
1 & 1 & 1 \\
3 & -4 & 2 \\
0 & 0 & 0
\end{bmatrix}
\]

Row operations yield:

\[
\begin{bmatrix}
1 & 0 & \frac{6}{7} \\
0 & 1 & \frac{1}{7} \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\frac{4}{7} & \frac{1}{7} & 0 \\
\frac{3}{7} & -\frac{1}{7} & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

And therefore there is no inverse.
Outline

1. Solving a Matrix Equation
2. The Inverse of a Matrix
3. Solving Systems of Equations
4. Conclusion
Using $A^{-1}$ to Solve a System of Equations

The main reason we are interested in matrix inverses is to solve a system of equations written in matrix form.

Solving a System of Equations

If $A$ is the matrix of coefficients for a system of equations, $X$ is the column vector containing the system variables, and $B$ is the column vector containing the constants, then:

$$AX = B \Rightarrow A^{-1}AX = A^{-1}B$$

$$\Rightarrow I_nX = A^{-1}B$$

$$\Rightarrow X = A^{-1}B$$

Caution:

A system of equations can only be solved in this way if it has a unique solution.
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$$\Rightarrow X = A^{-1}B$$

**Caution:**

A system of equations can only be solved in this way if it has a unique solution.
An Example

Example

Use a matrix equation to set-up and solve each system of equations given below.

1. \[
\begin{align*}
-x - 2y &= 1 \\
3x + 4y &= 3
\end{align*}
\]

2. \[
\begin{align*}
x + y - z &= 6 \\
3x - y &= 8 \\
2x - 3y + 4z &= -3
\end{align*}
\]
Example

Use a matrix equation to set-up and solve each system of equations given below.

1. \[
\begin{align*}
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x & + y - z = 6 \\
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1. \[
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-x - 2y &= 1 \\
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2. \[
\begin{align*}
x + y - z &= 6 \\
3x - y &= 8 \\
2x - 3y + 4z &= -3 \\
\end{align*}
\]
Reusing Results

One major advantage of solving a system of equations using a matrix equation is that if your matrix of coefficients $A$ stays the same, but your matrix $B$ changes, you can reuse most of your work.

**Example**

Solve each of the following systems of equations using the results from the last part of the previous question.

\[
\begin{align*}
\begin{cases}
    x + y - z &= 2 \\
    3x - y &= 1 \\
    2x - 3y + 4z &= 0
\end{cases}
\end{align*}
\]
One major advantage of solving a system of equations using a matrix equation is that if your matrix of coefficients $A$ stays the same, but your matrix $B$ changes, you can reuse most of your work.

**Example**

Solve each of the following systems of equations using the results from the last part of the previous question.

1. \[
\begin{align*}
    x + y - z &= 2 \\
    3x - y &= 1 \\
    2x - 3y + 4z &= 0
\end{align*}
\]

2. \[
\begin{align*}
    x + y - z &= 0 \\
    3x - y &= -14 \\
    2x - 3y + 4z &= -13
\end{align*}
\]
Reusing Results

One major advantage of solving a system of equations using a matrix equation is that if your matrix of coefficients $A$ stays the same, but your matrix $B$ changes, you can reuse most of your work.

Example

Solve each of the following systems of equations using the results from the last part of the previous question.

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    x + y - z &= 2 \\
    3x - y &= 1 \\
    2x - 3y + 4z &= 0
\end{align*}
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2. \[
\begin{align*}
    x + y - z &= 0 \\
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#### Things to Remember from Section 2-6

1. \( A^{-1}A = AA^{-1} = I_n \) for an \( n \times n \) matrix \( A \)

2. Find \( A^{-1} \) by reducing \([A \mid I_n]\) to \([I_n \mid A]\)

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