Outline

1. Solving Linear Programming Problems
2. Linear Programming Solution Procedure
3. Dealing with Unbounded Regions
4. Conclusion
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1. Solving Linear Programming Problems
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Solving a Linear Programming Problem

Recall that at the end of the last section, we found the corner points of the regions we graphed.

Example

Last time we set-up the following linear programming problem to determine how many batches of hot and mild salsa to make in order to maximize profit.

Objective: Maximize profit of $3x + 7y$

Constraints:

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\begin{align*}
10x + 8y &\leq 400 \\
x + 3y &\leq 100 \\
x &\geq 0 \\
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Finding the Optimal Solution

Example

The graph for the critical region is shown below, along with the graph of $3x + 7y = C$ for several different $Cs$.

As $C$ changes, the line $3x + 7y = C$ moves.

Unless it has the same slope as one of the boundary lines, it will first touch the region at a corner point.
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Thus, to find the optimal number of batches of hot and mild salsa to make, we need only check the corner points to see which produces the most profit.

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Completing The Example

The table above shows the corner points and their corresponding profits, allowing us to determine the optimal number of batches to make.
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Note that because this is a story problem, \(\frac{200}{11}\) and \(\frac{300}{11}\) are not possible answers. If you try (18, 27), which is close, you find a maximum profit of 243.
From this example, we can develop a general procedure for solving a linear programming problem.

To solve a linear programming problem:

- Graph the region given by the constraints.
- Find the corner points of the region.
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An Example with Multiple Solutions

Example

Solve the following linear programming problem.

Objective: Minimize $2x + 6y$

Constraints:

\[
\begin{align*}
x + 3y & \leq 30 \\
3x - 4y & \geq -27 \\
x & \geq 0 \\
y & \geq 0
\end{align*}
\]
Outline

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An Unbounded Region

Example

Solve the following linear programming problem.

Objective: Maximize \( 2x - y \)

Constraints:
\[
\begin{align*}
2x + 3y & \geq 6 \\
3x - 2y & \geq -6
\end{align*}
\]

\[
\begin{align*}
x & \geq 1 \\
y & \geq 1
\end{align*}
\]
Unbounded Regions

If the region formed by the constraints is unbounded, there is one more step to the solution process.

Solution Procedure

1. Graph the feasible set given by the system of equations.
2. Find the corner points of the region (if empty, no solution).
3. Check the objective function at each corner point.
4. If the region is bounded, the solution is the corner point with the min/max value. If two corner points have the same min/max value, there are infinitely many solutions.
5. If the region is unbounded, and your min/max value is found on a line to infinity, pick an extra point farther along that line and check it. If it is your new min/max point, then there is no solution. Otherwise, the original corner point is your solution.
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