MATH 105: Finite Mathematics
6-3: The Multiplication Principle

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Outline

1. Introduction
2. Examples
3. Conclusion
In the last section we counted the number of elements in combinations of sets with known sizes. But what if we aren’t told the size of a set to begin with?

**Example**

A value meal consists of your choice of one sandwich and one side dish from a menu with three sandwiches and four side dishes. How many possible meals are there?

If there are the same number of branches at each level, we can simply multiply.

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\frac{3}{\text{sandwich}} \cdot \frac{4}{\text{sides}} = \frac{12}{\text{meals}}
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Counting with Tree Diagrams

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Multiplication Principle

**Multiplication Principle of Counting**

If a task consists of a sequence of choices in which there are \( p \) selections for the first choice, \( q \) selections for the second choice, \( r \) selections for the third choice, and so on, then the task of making these selections can be done in

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p \cdot q \cdot r \cdot \ldots
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different ways.

**Note**

The number of selections available for the second choice can not depend on which first choice is made.
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Car Production

Example
A certain type of car can be purchased in any of five colors, with a manual or automatic transmission, and with any of three engine sizes. How many different car packages are available?

Note
It is still possible to draw a tree diagram in this example. It would, however, take more time than multiplying.
Example

A certain type of car can be purchased in any of five colors, with a manual or automatic transmission, and with any of three engine sizes. How many different car packages are available?

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License Plates

Example

Let $L$ be the set of Washington state license plates—three numbers followed by three letters. How many license plates are in the set?

Example

Now suppose that letters and digits a license plate may not be repeated. How many possible plates are there?
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Let $L$ be the set of Washington state license plates—three numbers followed by three letters. How many license plates are in the set?

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\[ c(L) = \frac{26 \cdot 26 \cdot 26}{\text{Letters}} \cdot \frac{10 \cdot 10 \cdot 10}{\text{Digits}} = 17,576,000 \]

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Now suppose that letters and digits a license plate may not be repeated. How many possible plates are there?

$$c(L) = \frac{26 \cdot 25 \cdot 24}{\text{Letters}} \cdot \frac{10 \cdot 9 \cdot 8}{\text{Digits}} = 11,232,000$$
Seating Arrangements

Example

There are 8 seats in the front row of a classroom, and 12 eager students wishing to fill them. In how many ways can these seats be assigned?

Note

This type of multiplication principle problem is very typical and will be given a special designation in the next section.
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\[12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 19,958,400\]

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Failure of the Multiplication Principle

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Recall our automobile which came in 5 colors, with 2 transmissions and 3 engine sizes. Suppose that the smallest engine only came with an automatic transmission. How many packages are now available?
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Important Concepts

Things to Remember from Section 6-3

1. Tree Diagrams can be very useful!
2. Multiplication Principle
3. There are cases where the Multiplication Principle does not work!
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The multiplication principle is the base for much of the counting we will do in this class. It is an important concept to know and practice.

Next time we will examine a specific type of Multiplication Principle problem which results in a counting rule called a “Permutation”.

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- Do Problem Sets 6-3 A,B
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