MATH 105: Finite Mathematics
9-4: Measures of Center

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Winter Quarter, 2006
Outline

1. The Mean
2. The Median
3. The Mode
4. Conclusion
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4. Conclusion
Measures of Center

In this section we will see various ways to measure the center of a set of data—that is, the “typical” value in the data set. The best known measure of center is often referred to as the average.

**Arithmetic Mean**

Let \( \{x_1, x_2, \ldots, x_n\} \) is a set of \( n \) real numbers. Then the arithmetic mean of the set is:

\[
\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}
\]

**Note:**

This is called the arithmetic mean because it involves addition. In contrast, the geometric mean is:

\[
\sqrt[n]{(x_1)(x_2)\ldots(x_n)}
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Computing the Mean

Example

Compute the arithmetic mean (mean) of each set of data.

1. \{65, 71, 75, 82, 82, 94\}
2. \{15, 65, 71, 75, 82, 82, 94\}

Note:
Note how big a difference the addition of one number can make in the mean. This shows that the mean is sensitive to outliers.
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\frac{65 + 71 + 75 + 82 + 82 + 94}{6} = \frac{469}{6} \approx 78.17
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2. \(\{15, 65, 71, 75, 82, 82, 94\}\)
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   \frac{15 + 65 + 71 + 75 + 82 + 82 + 94}{7} = \frac{484}{7} \approx 69.14
   \]

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Deviations from Mean

**Deviations**

If \( \{x_1, x_2, \ldots, x_n\} \) is a set of \( n \) data points and \( \bar{x} \) is the mean, then

\[
(x_1 - \bar{x}) + (x_2 - \bar{x}) + \ldots + (x_n - \bar{x}) = 0.
\]

**Example**

Verify this rule with \( \{65, 71, 75, 82, 82, 94\} \).
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\]

Example

Verify this rule with \( \{65, 71, 75, 82, 82, 94\} \).

<table>
<thead>
<tr>
<th>Value</th>
<th>Mean</th>
<th>Deviation ( (x - \bar{x}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>65</td>
<td>78.17</td>
<td>-13.17</td>
</tr>
<tr>
<td>71</td>
<td>78.17</td>
<td>-7.17</td>
</tr>
<tr>
<td>75</td>
<td>78.17</td>
<td>-3.17</td>
</tr>
<tr>
<td>82</td>
<td>78.17</td>
<td>3.83</td>
</tr>
<tr>
<td>82</td>
<td>78.17</td>
<td>3.83</td>
</tr>
<tr>
<td>94</td>
<td>78.17</td>
<td>15.83</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>
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Another Measure of Center

As we saw in the first two examples, the mean can be changed by a single value which is much smaller or larger than the rest of the data. To address this potential problem, we introduce another measure of center.

The Median

The median of a set of real numbers is found by arranging the numbers in order, from least to greatest, and then finding the middle value, if there is an odd number of values, or the mean of the two middle values if there is an even number of values.

Note:

As we will see in the next example, the median is not susceptible to outliers.
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Note:
As we will see in the next example, the median is not susceptible to outliers.
Computing the Median

Example

Find the median of each set of data.

1. \{71, 65, 82, 75, 94, 82\}
2. \{75, 15, 65, 82, 71, 94, 82\}
Computing the Median

Example

Find the median of each set of data.

1. \{71, 65, 82, 75, 94, 82\}

2. \{75, 15, 65, 82, 71, 94, 82\}
Computing the Median

Example

Find the median of each set of data.

1. \( \{71, 65, 82, 75, 94, 82\} \) Median: \( \frac{75 + 82}{2} = 78.5 \)

2. \( \{75, 15, 65, 82, 71, 94, 82\} \)
Computing the Median

Example

Find the median of each set of data.

1. \{71, 65, 82, 75, 94, 82\}  Median: \(\frac{75+82}{2} = 78.5\)

\[
\begin{array}{cccccc}
65 & 71 & 75 & 82 & 82 & 94 \\
\uparrow & \uparrow \\
\end{array}
\]

2. \{75, 15, 65, 82, 71, 94, 82\}
Computing the Median

Example

Find the median of each set of data.

1. \( \{71, 65, 82, 75, 94, 82\} \)  Median: \( \frac{75 + 82}{2} = 78.5 \)

   \[
   \begin{array}{cccccc}
   65 & 71 & 75 & 82 & 82 & 94 \\
   \uparrow & \uparrow \\
   \end{array}
   \]

2. \( \{75, 15, 65, 82, 71, 94, 82\} \)  Median: 75

   \[
   \begin{array}{cccccc}
   15 & 65 & 71 & 75 & 82 & 82 & 94 \\
   \uparrow \\
   \end{array}
   \]
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One Last Measure of Center

The last measure of center which we will examine is the Mode. The mode is unique because it will work both with numerical data and with descriptive data.

Mode

The mode of a set of real numbers (or any set) is the value which occurs most frequently, with frequency greater than one.

Note:

It is possible for a set of data to have no mode or more than one mode. If there are two modes, the set is called bimodal. If there are more than two, the data set is called multimodal.
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Computing the Mode

Example

Find the mode of each set of numbers.

1. \{65, 71, 75, 82, 82, 94\}
2. \{65, 71, 75, 82, 94\}
3. \{3, 3, 3, 5, 7, 7, 9, 9, 9, 10\}
Computing the Mode

Example

Find the mode of each set of numbers.

1. \{65, 71, 75, 82, 82, 94\}
2. \{65, 71, 75, 82, 94\}
3. \{3, 3, 3, 5, 7, 7, 9, 9, 9, 10\}
Finding the Mode

Example

Find the mode of each set of numbers.

1. \{65, 71, 75, 82, 82, 94\}
   The Mode is 82 (appears twice)

2. \{65, 71, 75, 82, 94\}

3. \{3, 3, 3, 5, 7, 7, 9, 9, 9, 10\}
Computing the Mode

Example

Find the mode of each set of numbers.

1. \{65, 71, 75, 82, 82, 94\}
   The Mode is 82 (appears twice)

2. \{65, 71, 75, 82, 94\}

3. \{3, 3, 3, 5, 7, 7, 9, 9, 9, 10\}
Computing the Mode

Example

Find the mode of each set of numbers.

1. \{65, 71, 75, 82, 82, 94\}
   
   The Mode is 82 (appears twice)

2. \{65, 71, 75, 82, 94\}
   
   no mode (each value appears once)

3. \{3, 3, 3, 5, 7, 7, 9, 9, 9, 10\}
Computing the Mode

Example

Find the mode of each set of numbers.

1. \{65, 71, 75, 82, 82, 94\}
   The Mode is 82 (appears twice)

2. \{65, 71, 75, 82, 94\}
   no mode (each value appears once)

3. \{3, 3, 3, 5, 7, 7, 9, 9, 9, 10\}
Computing the Mode

Example

Find the mode of each set of numbers.

1 {65, 71, 75, 82, 82, 94}
   The Mode is 82 (appears twice)

2 {65, 71, 75, 82, 94}
   no mode (each value appears once)

3 {3, 3, 3, 5, 7, 7, 9, 9, 9, 10}
   bimodal: 3 and 9 (both appear three times)
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Important Concepts

Things to Remember from Section 9-4

1. Mean: Sensitive to outliers, but every value contributes

2. Median: Not sensitive to outliers, depends only on middle value(s)

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Measuring the “typical” value of a set of data is only one part of the picture. It is also important to know how spread out from this center point the data is. In the next section we will see several measures of spread.

For next time

- Read section 9-5
Measuring the “typical” value of a set of data is only one part of the picture. It is also important to know how spread out from this center point the data is. In the next section we will see several measures of spread.

For next time
- Read section 9-5