MATH 112

Section 1.3: Recognizing Patterns

Section 1.4: Representation

Section 1.5: Reasoning and Proof

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Outline

1. Patterns
2. Representation
3. Reasoning and Proof
4. Venn Diagrams
5. Conclusion
As we saw in the lab this week, one important part of problem solving is looking for and recognizing patterns.

**Example**

Find the sum of the first 100 numbers.

**Methods for Finding Patterns**

How can the following methods be used to solve this problem?

1. Look for patterns in a simpler problem.
2. Look for patterns in the whole list of numbers.
3. Look for patterns in a geometric representation.
The Representation Standard

Representation is one of the NCTM standards used throughout the course.

NCTM Standard 10: Representation

Instructional programs from pre-kindergarten through eighth grade should enable all students to:

- create and use representations to organize, record, and communicate mathematical ideas.
- select, apply, and translate among mathematical representations to solve problems.
- use representations to model and interpret physical, social, and mathematical phenomena.
Often in our search for patterns, the hardest task to deciding how to arrange the data we collect to make those patterns as obvious as possible.

**Example**

How many ways can one make up $0.37 of postage using only $0.37, $0.15, $0.10, and $0.01 stamps?

As you search for patterns, think systematically. Come up with a way to collect your data or list possibilities so that you will not miss any of the options.
The NCTM Standard

Reasoning and Proof are an important part of mathematics. Logical thinking skills are more important than learning specific mathematical facts.

NCTM Standard 7: Reasoning and Proof

Instructional programs from pre-kindergarten through eighth grade should enable all students to:

- recognize reasoning and proof as fundamental aspects of mathematics.
- make and investigate mathematical conjectures.
- develop and evaluate mathematical arguments and proofs.
- select and use various types of reasoning and methods of proof as appropriate.
What is Reasoning?

Mathematics rely on words more than numbers. In section 1.5, we use words and thinking processes to solve problems.

Types of Reasoning

In this section we will consider the three types of reasoning.

- Intuitive Reasoning – Solving a problem with a “flash of inspiration.”
- Inductive Reasoning – Forming general conclusions by looking at patterns and specific examples.
- Deductive Reasoning – Working from general rules or known facts to specific conclusions.

Example

What type of reasoning was used to solve the dart board problem?
Using Inductive Reasoning

Example
What is the 10th term of the Fibonacci sequence, 1, 1, 2, 3, 5, . . .

Types of Sequences
There are other types of sequences:
- Geometric Sequences: \( a, ar, ar^2, ar^3, \ldots, ar^{n-1}, \ldots \)
- Arithmetic Sequences: \( a, a + d, a + 2d, a + 3d, \ldots, a + (n - 1)d, \ldots \)

Example
Find the 20th term of each sequence.
- 2, 6, 18, 54, . . .
- 1, 6, 11, 16, . . .
Using Deductive Reasoning

Deductive reasoning is using general rules of logic applied to specific. Let’s look at some of those rules of logic.

Conditional Statements

A conditional statement is an “if, then” statement in which an hypothesis implies a conclusion.

Example

The following is a conditional statement. Make the same statement in at least two different ways.

If I do my homework, then I will pass this class.
Equivalent Conditional Statements

Rewriting conditional statements can be tricky. Certain rewritings are “logically equivalent” (mean the same thing) and others are not.

Related Conditional Statements
If $p \rightarrow q$ is a conditional statement, then there are three related statements of special interest.

- Converse ($q \rightarrow p$)
- Inverse (not $p \rightarrow$ not $q$)
- Contrapositive (not $q \rightarrow$ not $p$)

Example
Find the three related forms of the conditional statement *If I sleep until noon, then I will miss my class*. Which of these are logically equivalent?
Deductive reasoning allows us to combine conditional statements to form arguments, or proofs.

**Modus Ponens**

If the conditional statement $p \rightarrow q$ is true, and the statement $p$ is true, then the statement $q$ is also true. This is called *affirming the hypothesis*.

**Modus Tollens**

If the conditional statement $p \rightarrow q$ is true, and the statement $q$ is false, then the statement $p$ is also false. This is called *denying the conclusion*.
Logical Fallacies

Deductive reasoning can sometimes be confusing. Avoid these incorrect forms of argument.

Affirming the Conclusion
If the conditional statement $p \rightarrow q$ is true, and the statement $q$ is true, then the statement $p$ is true.

Denying the Hypothesis
If the conditional statement $p \rightarrow q$ is true, and the statement $p$ is false, then the statement $q$ is false.
Visualizing Conditional Statements

One method of representing conditional statements and sets is with Venn Diagrams.

- Universal Set
- Set $A$
- Set $B$
- $A \cap B$
Using Venn Diagrams

Example
Use Venn Diagrams to determine the validity of the following conditional statement and its three related statements.

If a figure is a square, then it has four sides.

Example
Use Venn Diagrams to determine if each argument is valid.

1. All professional athletes are rich. All rock stars are rich. Therefore, some rock stars are professional athletes.
2. All bunnies are skittish. Some skittish animals have fur. All cows have fur. Therefore, cows are skittish animals.
3. All rectangles are quadrilaterals. All quadrilaterals are polygons. Therefore, all rectangles are polygons.
Patterns
Representation
Reasoning and Proof
Venn Diagrams
Conclusion

Important Concepts

Things to Remember from Sections 1.3 through 1.5

1. Strategies for Finding Patterns
2. Systematic Representation of Solutions
3. Three Types of Reasoning
4. Finding Terms in a Sequence
5. Conditional Statements and Related Forms
6. Logical Arguments
7. Venn Diagrams