MATH 112
Section 4.1: Divisibility

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Outline

1. Introduction and Terminology
2. Examining Odds and Evens
3. Divisibility Rules
4. Conclusion
What is Divisibility?

Now that we have looked at each of the four basic operations of arithmetic, it is time to start examining relationships between numbers in more depth. One of the basic relationships is that of divisibility.

**Divisibility**

If $a$ and $b$ are two whole numbers with $b \neq 0$, then $a$ divides $b$ ($a|b$) if there is some whole number $t$ with $at = b$.

**Example**

One way in which divisibility is commonly used is in the definition of odd and even numbers.

- Even numbers are numbers which are divisible by 2.
- Odd numbers are numbers which are not divisible by 2.
There are a few terms we need to become familiar with in our discussion of divisibility.

### Divisibility Terminology

If $a|b$ as in the previous definition, then we can say the following:

- $a$ is a factor of $b$
- $b$ is a multiple of $a$
- $b$ is divisible by $a$
- $a$ is a divisor of $b$

### Example

- 3 is a factor of 24
- 12 is a multiple of 6
- 25 is divisible by 5
- 7 is a divisor of 14
Odd and Even Revisited

Let's consider odd and even numbers again. What can we say about them?

**Properties of Even Numbers**

Even numbers have the following properties.

- 2 is a factor of each even number
- Even numbers are divisible by 2
- Even numbers are sums of two equal numbers

**Properties of Odd Numbers**

Odd numbers have the following properties.

- 2 is not a factor
- Remainder of one when divided by 2
Cuisinare rods can be used to model divisibility problems. In particular, they can be used to tell if a number is odd or even.

**Modeling Even Numbers**

The red cuisinare rod represents 2. A number is even if it can be represented by a train of red cuisinare rods.

**Example**

Use cuisinare rods to find the first 5 even numbers.
Patterns in Odd and Even Numbers

Before we explore further the list of numbers which are even, let's consider some of the patterns which we can find.

**Even/Odd Numbers and Addition**

The sum of any two odd numbers is even. The sum of any two even numbers is even. The sum of an even and an odd number is odd.

**Even/Odd Numbers and Algebra**

Any even number can be represented as $2n$ for some whole number $n$. Any odd number can be represented as $2n + 1$ for some whole number $n$.

**Example**

Using the algebraic properties of odd and even numbers stated above, prove the addition properties.
The first five even numbers found in a previous slide were 0, 2, 4, 6, and 8. What happens as we continue on?

**Example**

The first ten even numbers are 0, 2, 4, 6, 8, 10, 12, 14, 16, and 18. Do you notice a pattern here?

**Divisibility Test for Two**

A number is divisible by two if the last digit is divisible by two. That is, if it ends in a 0, 2, 4, 6, or 8.

**Why Does This Work?**

Try testing a few numbers represented with base ten blocks. What happens to each flat and long in the representation?
A General Divisibility Rule

As we start looking for more divisibility rules in base ten, it will be helpful to have a general divisibility rule.

**A Divisibility Rule**

For whole numbers $a$, $b$, and $c$, if $a|b$ and $a|c$ then $a|(b + c)$.

**Does the Converse Work?**

The converse of this statement would say “if $a|(b + c)$ then $a|b$ and $a|c$. Is this true?

**Example**

- $3|6$ and $3|9$ so $3|(3 + 9)$
- $3|9$ but $9 = 4 + 5$ and $3 \nmid 4$ and $3 \nmid 5$
Divisibility rules for 10 and 5 are similar, and you are probably already familiar with them.

**Divisibility by 5**
A whole number is divisible by 5 if the last digit is a 5 or 0.

**Divisibility by 10**
A whole number is divisible by 10 if the last digit is a 0.

**Example**
Using base 10 blocks as a model, show why these divisibility rules work.
Another pair of divisibility rules which you may have seen before are rules for 3 and 9. They are a bit more complicated to check.

**Divisibility by 3**

A whole number is divisible by 3 if the sum of all the digits is divisible by 3.

**Divisibility by 9**

A whole number is divisible by 9 if the sum of all the digits is divisible by 9.

**Example**

Use base 10 blocks to model why these divisibility rules work.
Introduction and Terminology

Examining Odds and Evens

Divisibility Rules

Conclusion

Base 10 Divisibility Rules for 4, 8, and $2^n$

In the divisibility rule for 2, we only had to look at the last digit because longs, flats, and all bigger powers of 10 can be divided into two pieces.

**Divisibility by 4**

A whole number is divisible by 4 if the last two digits form a two digit number divisible by 4.

**Divisibility by 8**

A whole number is divisible by 8 if the last three digits form a three digit number divisible by 8.

**Divisibility by $2^n$**

A whole number is divisible by $2^n$ if the last $n$ digits form a $n$-digit number divisible by $2^n$. 
Base 10 Divisibility Rules for 6 and 12

Our last two divisibility rules are interesting because we make use of previous rules to construct new ones.

**Divisibility by 6**
A whole number is divisible by 6 if it is both divisible by 2 and divisible by 3.

**Divisibility by 12**
A whole number is divisible by 12 if it is both divisible by 3 and divisible by 4.

**Another Divisibility Rule?**
Is this a divisibility test for 12? “A whole number is divisible by 12 if it is both divisible by 2 and by 6”
Important Concepts

Things to Remember from Section 4.1

1. Divisibility Terms and Definitions
2. Properties of Odd and Even Numbers
3. The General Divisibility Rule: if $a|b$ and $a|c$ then $a|(b + c)$.
4. Rules and Justification for Base 10 Divisibility by: 2, 3, 4, 5, 6, 8, 9, and 10.