MATH 112

2.1: Sets

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Outline

1. Defining Sets
2. Subsets and Combining Sets
3. Sets and Counting
4. Conclusion
What is a Set?

In Wednesday’s lab we started to work with sets. Let’s take a moment to think about what a set really is.

**A Set Is...**

A set is a well-defined collection of objects.

Well-defined means that it is absolutely clear what is in the set and what is not.

**Example**

Which of the following sets are well-defined?

1. The set of all math 112 students
2. The set of all tall students
3. The set of hard classes
4. The set of math classes
There are several ways which we can define a set.

**Ways to Write a Set**

- **Words**
  We use English to describe the contents of the set ($E$ is the set of all even whole numbers).

- **Lists**
  We write out all elements of the set ($E = \{0, 2, 4, 6, 8, \ldots\}$).

- **Set-Builder Notation**
  We use mathematical symbols to write a rule for membership in the set ($E = \{2x \mid x \text{ is an integer}\}$).

**Example**

Write the set of prime numbers less than 20 using all three methods.
One important concept involving sets is that of a subset.

**Subsets**

A subset of a set $A$ is a set $B$ all of whose elements are also in $A$.

**Example**

Which of the following sets are subsets of which others?

- $\mathbb{Z} = \text{the integers}$
- $E = \text{the even integers}$
- $O = \text{the odd integers}$
- $F = \text{integer multiples of 4}$
Counting Subsets

Any given set can have many different subsets. This is demonstrated in the following example.

**Example**

A student has the choice to take any subset of the classes \{ math, chemistry, history, bible \}. How many ways can she choose classes from this set?

**The Empty Set**

The empty set (\(\emptyset\)) contains no elements. It is a subset of any set (including itself).

**Equal and Equivalent Sets**

Two sets are **equal** if they have exactly the same elements. Two sets are **equivalent** if they have the same number of elements.
Combining Sets

There are several ways to combine sets to create new sets.

**Combining Sets**

If $U$ is a *universal set* and $A$ and $B$ are subsets of $U$, then we can combine sets as follows.

- **Intersection** – $A \cap B = \text{members of both } A \text{ and } B$
- **Union** – $A \cup B = \text{members of either } A \text{ or } B$
- **Complement** – $\overline{A} = \text{elements of } U \text{ not in } A$

**Example**

Let $U$ be the set of WWC students, $R$ the set of dorm students, and $M$ the set of math 112 students. Find each set below.

<table>
<thead>
<tr>
<th>$R \cap M$</th>
<th>$R \cup M$</th>
<th>$\overline{R}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R \cup M$</td>
<td>$R \cap M$</td>
<td>$\overline{R \cap M}$</td>
</tr>
</tbody>
</table>
Counting with Sets

In the lab, we used a Venn Diagram to help us count students with various pets. Use this technique to solve the following problems.

Example

You ask 150 people what they do in their spare time. 120 said that they watch TV, 70 play sports, and 60 do both. How many people neither watch TV nor play sports?

Example

In a mathematics class with 250 students, 100 are also taking history, 150 are taking Bible, and 200 are in an English composition class. Of those students in the mathematics class who are also taking English, 25 are taking neither history nor Bible, 75 are taking both History and Bible, and 25 are taking Bible but not history. How many students are taking mathematics and history but neither Bible nor English?
Important Concepts

Things to Remember from Section 2.1

1. Three ways to define sets
2. Working with subsets
3. Combining sets
4. Using sets for counting problems