MATH 112
Section 3.4: Understanding Division

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Outline

1. Interpretations of Division
2. Properties of Division
3. Division Algorithms
4. Mental Division Methods
5. Conclusion
Division vs. Multiplication

Just like subtraction was the opposite of addition, division undoes multiplication. Before we start working division problems, we need some terminology.

**Multiplication and Division**

For whole numbers $a$, $b$, and $n$ where $n$ is not zero,

$$ a \div n = b \quad \text{exactly when} \quad b \times n = a $$

**Parts of a Division Process**

If $a \div n = b$ then

- $a$ is called the dividend
- $n$ is called the divisor
- $b$ is called the quotient
Partitioning Model

One of the easiest models of division for many people to understand is called the partitioning model. Consider the following example.

Example

You have 16 cookies to share with your three friends. How many cookies will you each get if you share them equally?

Divide the 16 cookies into four piles, one cookie at a time. When done, there are 4 cookies in each pile, so the answer is four.

The Partition Model

To divide a whole number $a$ by another whole number $b$, partition $a$ into $b$ equal portions (where possible). The number in each portion is the quotient, and anything left over is the remainder.
Repeated Subtraction Model

A model that ties in well with the “repeated addition” model of multiplication is the “repeated subtraction” model for division.

Example

You are making miniature apple pies. Each pie requires 6 ounces of apples. You have 18 total ounces of apples. How many pies can you make?

Take 6 away from 18 repeatedly until you no longer have 6 to take away. Since we can take away 3 groups of six, the answer is 3.

The Repeated Subtraction Model

To divide a whole number \(a\) by another whole number \(b\), divide the quantity \(a\) into portions of size \(b\). The number of resulting portions is the quotient, and anything left over is the remainder.
Comparing the Two Models

To see how the two models we have examined differ, let’s solve the following problem in two different ways.

**Example**

Divide the number 10 by 2.

Start with a train of 10 blocks. Divide it into two equal trains. Each train is five units long, so the quotient is five.

Using a train of 10 blocks as our starting point, divide the train into smaller trains of length two. Since we get five trains of length two, the quotient is five.

**Other Models**

Other models of division include the “missing factor” and “number line” models.
Properties of the Four Operations

The following table describes the properties of each of the four operations we have examined.

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<th>Commutative</th>
<th>Associative</th>
<th>Identity</th>
<th>Inverses</th>
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</table>
In our first slide, we insisted that if we want to divide $a$ by $b$, then $b$ could not be zero. Why is this?

**Dividing by Zero**

If we could divide by zero, then we could write $a \div 0 = c$. By our definition, this would mean $0 \times c = a$. But $0 \times c = 0$ no matter what $c$ is.

**Trying to Divide by Zero**

What happens if we try to divide by zero? Using a calculator, try dividing the same dividend by smaller and smaller divisors. For example, $10 \div 10$, $10 \div 1$, $10 \div 0.1$, $10 \div 0.01$ and so on.
Division can be modeled using base 10 blocks just as addition, subtraction, and division. We can use either the partition or repeated subtraction model to accomplish this.

Example
Use base 10 blocks to model each division problem.

\[
174 \div 3 = 58 \quad 337 \div 4 = 84 \, \text{R}1
\]

Example
Use standard long division to divide 337 by 4 and tie it together with the base 10 block model above.
As with previous operations, we assume that you know the “standard” long division algorithm.

Example

Use the standard division algorithm to solve the following problem:

$$14948 \div 71$$
Another interesting algorithm makes use of “guess-and-check” methods and breaks division problems down into smaller pieces.

**Example**

Use scaffolding to solve each division problem.

\[ 576 \div 8 \]

\[ 6371 \div 24 \]
Algorithms for Mental Division

There are a few tricks that can make division easier to do mentally.

Example

Find each product mentally as quickly as possible.

1. $6000 \div 20$
2. $20000 \div 400$
3. $152 \div 8$
4. $882 \div 9$
Important Concepts

Things to Remember from Section 3.4

1. Interpretations of division
2. Properties of division
3. Alternative division algorithms
4. Mental division techniques