MATH 112
Section 4.2: Primes and Composites

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What is a Prime Number?

As we saw in the locker problem, the number of factors which a number has can be important to the structure of the number. One special kind of number is a prime number.

**Prime Number**

A prime number $p$ is a number whose only factors are 1 and itself.

**Composite Number**

A number which is not prime is called a composite number.

**Example**

Is the number 169 prime? How can you find out?
The Sieve of Eratosthenes

One method for finding prime numbers is called the sieve of
Eratosthenes. This is named for the Greek mathematician and
philosopher who first discovered it.

The Sieve

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Example

To find prime numbers, circle the first unmarked number and then
cross off all multiples of that number. Repeat this process.
The procedure for finding primes seen on the last slide leads us to a question. Is there a largest prime number?

Example
Is there a largest prime number? How could you tell?

- Suppose there were a largest prime $p$
- Consider the product $X = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot P + 1$
- Is $X$ divisible by any number $\leq P$?
- Then $X$ must be prime and $> P$
Prime numbers are important because they form the building blocks for every whole number.

**Fundamental Theorem of Arithmetic**

Every whole number greater than one can be written uniquely as a product of prime numbers listed in increasing order.

**Things to Notice**

- There is only one final factorization into primes
- Composite numbers are products of more than one prime
- Prime numbers fit in since they are themselves prime
Factoring into Products of Prime

Although there is only one way to write a number as a product of prime numbers, there are usually several different ways to find that product.

**Example**

Write each number below as a product of primes. For each number, find these products in two different ways.

1. 24
2. 352
Classification by Factor Sums

Prime and Composite numbers are not the only categories into which we can group numbers. Another way to classify numbers is by the sum of their factors.

**Perfect Numbers**
A whole number is perfect if it is equal to the sum of its proper divisors.

**Abundant Numbers**
A whole number is abundant if it is less than the sum of its proper divisors.

**Deficient Numbers**
A whole number is deficient if it is greater than the sum of its proper divisors.
Classifying Numbers

How do these classifications relate to prime numbers?

Perfect, Abundant, Deficient Numbers and Primes

- Can a prime number be abundant or perfect?
- Can a the product of two primes be abundant or perfect?

Example

For each whole number from 1 to 30, determine if the number is Deficient, Perfect, or Abundant.
Important Concepts

Things to Remember from Section 4.2

1. The definition of a prime and composite number
2. How to write a number as a product of primes
3. Identifying perfect, abundant, and deficient numbers