MATH 113
Section 10.2: Area and Perimeter

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Outline

1. Perimeter
2. Area
3. Area and Arc Length of Circular Sectors
4. Conclusion
In this section we will examine two measurements common to two-dimensional figures.

**Perimeter**

Perimeter is the distance around an object. In a polygon, this can be determined by adding the length of the sides.

**Example**

Find a formula for the perimeter of each figure:

1. A square of side length $s$.
2. An isosceles triangle with base length $b$ and side length $s$.
3. A rectangle of width $w$ and height $h$. 
Perimeter and the Pythagorean Theorem

One of the most well-known theorems of elementary geometry has to do with side lengths (perimeter) for certain types of triangles.

**Pythagorean Theorem**

In a right triangle as shown to the right,

\[ a^2 + b^2 = c^2 \]

**Proof of the Pythagorean Theorem**
A Pythagorean Example

While it may seem like the pythagorean theorem has limited applicability, it can be useful in many different situations.

Example

Use the pythagorean theorem to determine the length of the lake pictured below.
The Pythagorean theorem is also useful for finding the perimeter of polygons as they can often be broken into right triangles.

Example

Find the perimeter if each geoboard figure below.
Definition of Area

While perimeter is a measure of length around a figure, area is the measure of the enclosed region of the figure.

Area

Area is a physical quantity expressing the size of a part of a surface.

Example

Find a formula for the area of each figure:

1. A square of side length $s$.
2. An isosceles triangle with base length $b$ and side length $s$.
3. A rectangle of width $w$ and height $h$. 
Area of Irregular Shapes

Area formulas for polygons are relatively simple as we can break these figures down into squares or triangles. How do we find the area of irregularly shaped figures?

Example

How could one measure this irregularly shape?

1. Use a grid or graph paper.
2. Use triangles and rectangles to estimate.
3. Use trapezoids to estimate.
4. Find the area of a rectangle with the same perimeter.
5. Weigh it.

Be Careful!

Does number 4 above really work?
Relation Between Area and Perimeter

While there is a relationship between area and perimeter (which we have explored in lab), figures with the same perimeter need not have the same area.

**Example**

Find the area and perimeter of rectangles with the following lengths and widths.

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>20</td>
</tr>
<tr>
<td>70</td>
<td>30</td>
</tr>
<tr>
<td>60</td>
<td>40</td>
</tr>
</tbody>
</table>
Square Units

It is important to remember that since area is a two dimensional measurement, the units will be different.

Example

How many feet are there in a square yard?

Example

Give the number of square yards in a rectangle that is 4 feet by 8 feet.

Dimensional Analysis

You can keep track of the correct units in an area measurement by multiplying the linear units together. For example, a rectangle which is \(w\) feet by \(l\) feet will have area \((w \times l)\) feet \(\times\) feet \(= (w \times l)\) square feet.
Circumference and Arc Length

The perimeter of a circle has a special name.

Circumference

The circumference (perimeter) of a circle is given by the formula

\[ C = \pi d = 2\pi r \]

What is \( \pi \)?

\( \pi \) is defined to be the ratio of the circumference of a circle to its diameter. This is constant for all circles. That is, \( \pi = \frac{C}{d} \). Solving this for \( C \) yields the above equation.

Example

Find the circumference of a circle of radius 4 and a circle of diameter 7.
An Arc Length Example

Using proportions, we can find the arc length, or perimeter, of a piece of a circle.

**Example**

Find the arc length of an arc of 40° from a circle of radius 6.

**Solution**

- Find the circumference of the entire circle.
- The first ratio is the arc length to this circumference.
- The second ratio is the angle measure to the entire 360° in the circle.
- Solving this proportion yields the arc length.
### Calculating the Circumference of the Earth

The following is the procedure Eratosthenes used to calculate the circumference of the earth.

- On a certain day the sun was directly overhead in the town square of Syene (He noticed that the sun shown straight down a well).

- On that same day, in the town of Alexandria 5000 stadia (489 miles) to the north, the sun made an angle of $7.2^\circ$.

- Using alternate interior angles, Eratosthenes determined that the central angle of the circular sector between the two cities is $7.2^\circ$.

- Using ratios, he then computed the circumference.

- His computed value of 24,450.00 miles is within 2% of the currently held value of 24,859.82 miles.
The area of a circle is a more difficult measurement to justify. You worked on this in your lab.

**Area of a Circle**

The area of a circle of radius $r$ is given by $A = \pi r^2$.

**Example**

Using the same techniques as seen in the circumference example, find the area of a $40^\circ$ circular sector.

**Example**

A pizza of diameter 16” has been cut into 12 equal slices. How much area is in each slice?
Important Concepts

Things to Remember from Section 10.2

1. Computing the perimeter of polygons.
2. Using the Pythagorean theorem to compute a perimeter.
3. Finding the area of a polygon.
4. Estimating the area of an irregular figure.
5. Finding area and arc length of circles.