Outline

1. Defining and Interpreting Percents
2. Percents and Change
3. Interest
4. Conclusion
What are Percents?

The term “percent” is from the Latin “per centum” which translates “per one hundred.”

**Percents**

Percents are related to fractions as follows. A percent (written $A\%$) stands for $\frac{A}{100}$.

**Example**

Write 30%, 112%, and 0.5% as both fractions and decimals.

**Example**

Write $\frac{14}{100}$, 0.003, and $\frac{5}{4}$ as percents.
Once we have learned how to express percents as fractions, we are able to use proportions to solve problems related to percents.

Example
You score 80% on a 20-point paper. What is your raw score on the paper?

Example
You score $\frac{10}{12}$ on a homework assignment. To what percent does your score translate?
Applications of Percents

While scores on exams and assignments are a big part of your current life, there are other important ways in which percents are useful in the real world.

- Weather Forecasting (30% chance of rain)
- Probability (0.01% chance of being struck by lightening)
- Rates of Change (annual inflation of 3%)
- Interest Rates (CD earns 5.5% APR)
- others?
Percents and the Whole

In percent problems, the whole is always 100%. So you must identify the whole in order to solve such problems.

Example

Property taxes are assessed at a rate of 1.2%. How much property tax would be due on a $142,500 home?

Estimated Solution

This is close to 1% which is $\frac{1}{100}$. So, divide the whole, $142,500, by 100 to get $1,425.

Computed Solution

Solve the proportion $\frac{1.2}{100} = \frac{t}{142,500}$ or simply multiply $.012 \times 142,500$ to get $1710$. 
Since percents are fractions with denominators of 100, the same modeling techniques as used with fractions will still work.

**Example**

Banks usually require that no more than 28% of a household’s monthly income be spent on a mortgage payment. How much must a household make to afford a $1200 monthly payment?

- Use a $10 \times 10$ grid to model 100% and shade in 28%.
- The shaded area also represents $1200$.
- Determine how much each box is worth.
- Now how much is the whole?

**Quicker Solution**

This can also be solved using the proportion $\frac{28}{100} = \frac{1200}{x}$. 


Types of Change

Using percents to describe change can be quite confusing. We need to be sure we understand how the percents are being used.

Example

A city’s 1.2% property tax rate will see a 1% increase in the new year. What will the new rate be?

Additive Change

If the change is additive, we are adding 1% to 1.2% so the new rate is 2.2%.

\[
\frac{101}{100} \times 1.2\% = \frac{100}{100} \times 1.2\% + \frac{1}{100} \times 1.2\% = 1.2\% + .012\% = 1.32\%
\]

Multiplicative Change

If the change is multiplicative, the new rate is:

\[
\frac{101}{100} \times 1.2\% = \frac{100}{100} \times 1.2\% + \frac{1}{100} \times 1.2\% = 1.2\% + .012\% = 1.32\%.
\]
Percents and Fairness

Example
A state experiences a budget shortfall. To raise the missing revenue they decide to raise everybody’s income taxes by $1000 a year. Is this fair?

Compare the percent increase in tax for somebody paying $2000 a year to somebody paying $18,000 a year.

Example
Instead the state decides to raise everybody’s taxes by 10% a year. Is this any better? How do the two taxpayers above fair?

Cutting Taxes
What about tax cuts? Is it fairer to cut taxes by set dollar amounts or by percents?
Percent Increases/Decreases

Another confusing aspect of percents and change is that the meaning of a percent increase or decrease depends on the whole.

Example

A local clothing store marks its clothing up 32%. In other words, they charge 32% more for an item than they paid their supplier.

1. If you purchase a $100 outfit, how much profit did they make?
2. During a sale, they mark a certain item down 32% and claim they are selling it below cost. Could this be true?

1. Solve $x + 0.32x = 100$
2. Solve $100 - 0.32(100) = x$
Simple Interest

One important use of percents is in calculating interest. This can be interest paid to you on your savings or interest you pay for loans.

Simple Interest

The formula for simple interest is:

\[ A = P (1 + r \times t) \]

where:

- \( P \) = principal (original amount)
- \( r \) = interest rate (i.e. 5%)
- \( t \) = number of years

Example

You put your $1000 tax refund in a CD paying 4% a year in simple interest. At the end of one year, how much do you have?
Compounded Interest

With compounded interest, you are paid a percent of what has accumulated in the account at regular time periods.

**Compounded Interest**

The formula for compounded interest is:

\[ A = P \left(1 + \frac{r}{n}\right)^{t \times n} \]

- \( P \) = principal
- \( r \) = interest rate
- \( t \) = number of years
- \( n \) = number of times per year interest is compounded

**Example**

Suppose that the $1000 was put into a 4% CD which compounded interest monthly. How much would you have after one year?
Important Concepts

Things to Remember from Section 6.2

1. Relating percents to fractions and decimals
2. Interpreting percents with models or in story problems
3. Finding percent changes
4. Working with percents and interest problems