Outline

1. What is Symmetry

2. Types of Symmetry
   - Reflective Symmetry
   - Rotational Symmetry
   - Translational Symmetry

3. Tessellations

4. Conclusion
Symmetry

Symmetry is an important aspect of geometry which can be found in every day life.

Example

Name several examples of symmetry seen in your daily life.

Symmetry and Nature

Symmetry is seen at every level of nature.

- molecular level
- in plants, animals and people
- galactic level
Symmetry Defined

While we may have an intuitive sense of what symmetry, it is still important to formalize a definition.

**Symmetry**
Symmetry is a transformation which places the object directly on top of itself.

Is this definition what you expected? How does it match up with your intuitive sense of symmetry?

**Symmetry and Congruence**
Since we wish to place an object directly on top of itself in a symmetric transformation, we will need to use congruence transformations to achieve symmetry.
Identifying Symmetry

With the definition of symmetry in mind, let’s consider several different figures and try to describe the symmetries we see.

Example

Describe the symmetry within each figure.
The first type of symmetry we will examine is reflective symmetry.

**Reflective Symmetry**
Reflective symmetry is achieved when a figure undergoes a reflection transformation and ends up in the same position and orientation.

**Example**
Determine if each of the following figures has reflective symmetry and find all lines of reflection producing symmetry.

- An equilateral triangle
- An isosceles triangle
- A square
- A kite
Reflective symmetry is often taught of as a desirable quality in facial features. Consider the following images.
Rotational Symmetry

Definition and Examples

All figures have $360^\circ$ rotational symmetry. We are interested in figures which can be rotated just a fraction of this amount.

Rotational Symmetry

A figure is said to have rotational symmetry if it can be rotated less than a full $360^\circ$ about some point and end up in the same position and orientation.

Example

Which of the following figures have rotational symmetry, and what is the smallest rotation resulting in this symmetry?

- Equilateral triangle
- Isosceles triangle with short base
- Kite with non-right angles
- A rectangle which is not a square
- A square
Rotational Symmetry and Regular Polygons

There is a relationship standard relationship between regular polygons and their rotational symmetries.

Example

Describe the rotational symmetries of the following regular polygons.

![Regular polygons]

Make A Guess

What sort of rotational symmetry do you think a regular $n$-gon will have? What about a circle?
The last sort of symmetry we will examine is that achieved by translations.

**Translational Symmetry**

A figure has translational symmetry if it can be translated in one direction and results in the same figure.

**Example**

Can you think of a figure with translational symmetry? What properties would such a figure have to have?
One area in which translational symmetry (as well as other types) is found is in border strip patterns.

**Example**

Describe the symmetries found in each of the following strip patterns.

a. ![Pattern Image](image1)
b. ![Pattern Image](image2)
c. ![Pattern Image](image3)
d. ![Pattern Image](image4)
e. ![Pattern Image](image5)
f. ![Pattern Image](image6)
Types of Symmetry in Strip Patterns

You book defines the following types of symmetry. Can you match these classifications up with the patterns we saw in the last slide?

<table>
<thead>
<tr>
<th>Vertical Line</th>
<th>Horizontal Line</th>
<th>(\frac{1}{2})-turn</th>
<th>Glide</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>ll</td>
</tr>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
<td>ml</td>
</tr>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
<td>lm</td>
</tr>
<tr>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
<td>mm</td>
</tr>
<tr>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
<td>lg</td>
</tr>
<tr>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
<td>l2</td>
</tr>
<tr>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
<td>mg</td>
</tr>
</tbody>
</table>

**Codes**

In the table above, m represents a line symmetry, g a glide symmetry, 2 a twofold turn symmetry, and l no symmetry.
What is a Tessellation

When covering a floor with a pattern, you want to pick a shape or shapes which will cover the whole floor without leaving gaps or overlapping.

**Tessellations**

We say that a figure or combination of figures tessellates the plane if a regular repetition of the figure or figures covers the plane so that there are no gaps and no overlapping of figures.

**Example**

M.C. Escher is famous for combining geometric concepts such as symmetry and tessellations with art. Consider some of the tessellations found on the official M.C. Escher website.

http://www.mcescher.com/
Tessellating Regular Polygons

Our tessellations will be simpler than Escher’s, but we still need to think carefully about what can and can’t be tessellated.

Example

Which of the following figures can be Tessellated?

- A triangle (what types?)
- A square
- A hexagon

In general, what types of regular polygons will tessellate? Why?
Important Concepts

Things to Remember from Section 9.2

1. Types of Symmetry
   - Reflective
   - Rotational
   - Translational

2. Symmetry in Strip Patterns

3. Tessellations