MATH 113
Section 9.2: Topology

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Geometric Topology

Our last kind of geometric transformation focuses on identifying what remains constant when we distort a figure. For this reason it is sometimes called “rubber sheet” geometry.

Example

Each figure below is topologically equivalent to exactly one other figure. That is, we can distort each figure to get exactly one of the other figures.

Topologically Equivalent

Two shapes are topologically equivalent if one can be turned into the other by stretching, shrinking, bending and/or twisting.
Since we can not cut or tear figures as we distort them, the number of “holes” in a figure becomes important as do the number of “legs”.

Example

Each numeral in the Hindu Arabic numeration system is topologically equivalent to exactly one of the following shapes. Determine which shapes go with which numerals.

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</table>
As young children develop, they are able to draw pictures which accurately represent certain relationships even if they do not accurately represent an object.

**Relationships**

Children's drawings often preserve:

- **Proximity** – while distances may not be accurate, what is nearer in an actual object is generally nearer in their drawing.
- **Separation** – what is separated and what is connected is usually preserved.
- **Order** – the relative order of objects is usually accurate.
- **Enclosure** – detains are either “inside” or “outside” of the appropriate region.
Examples

The examples below illustrate how these four relationships: proximity, separation, order, and enclosure.

Example

The three diagrams below are topologically equivalent.

Example

To a geometric topologist, a coffee cup and a donut are the same.
A Motivating Example

The Seven Bridges of Königsberg is a famous mathematics problem which will help us introduce the concept of a network.

Example

The city of Königsberg, Prussia (now Kaliningrad, Russia) is set on the Pregel River, and included two large islands which were connected to each other and the mainland by seven bridges. Around 1750 the prosperous and educated townspeople are said to have walked about on Sundays trying to determine if it is possible to plan a route that crosses each bridge exactly once and came back to the same starting point.
Königsberg Solution Part I

How does this problem relate to topology? The shape of a graph may be distorted in any way without changing the graph itself, so long as the links between nodes are unchanged.

Example

Networks

A network is a collection of points called the vertices and curves connecting those points called the edges.

As the Königsberg problem points out, it does not matter whether the edges are straight or curved, or how the nodes are arranged.
Königsberg Solution, Part II

The Königsberg bridge problem was solved by the mathematician Leonhard Euler who realized that the solution depended on the number of edges going into and out of each node.

**Degree**

The degree of a node is the number of edges touching it.

**Example**

In the Königsberg bridge graph, three nodes have degree 3 and one has degree 5. Euler proved that a circuit of the desired form is possible if and only if there are no nodes of odd degree. Such a walk is called an Eulerian circuit or an Euler tour. Since the graph corresponding to Königsberg has four nodes of odd degree, it cannot have an Eulerian circuit.
An Alternative Question

While the Königsberg bridge problem insisted that we start and end at the same point, we can relax this as follows.

Traversing a Network

A network is traversable if it is possible to travel each edge exactly once.

Traversable Networks

A network is traversable if all of its nodes have even degree or if it has exactly two nodes with odd degree.

Example

Which of the following networks are traversable?
Other Network Problems

Networks are useful in solving other types of problems as well.

A Coloring Problem

How many colors are needed to shade the following “map” so that no two adjacent regions have the same color?

A Scheduling Problem

Four students wish to take three classes as shown. How many different time slots are required for the classes to accommodate this schedule?

<table>
<thead>
<tr>
<th></th>
<th>Class 1</th>
<th>Class 2</th>
<th>Class 3</th>
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</thead>
<tbody>
<tr>
<td>Student 1</td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Student 2</td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Student 3</td>
<td>x</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student 4</td>
<td>x</td>
<td></td>
<td>x</td>
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</tbody>
</table>
Important Concepts

Things to Remember from Section 9.4

1. Identification of Topological Equivalent Figures
2. Traversable Networks
3. Using Networks to Solve Problems