MATH 113

Section 10.1: Systems of Measurement

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Outline

1. Measures of Time
2. Measures of Length
3. Measures of Weight/Mass
4. Measures of Volume
5. Measures of Temperature and Angles
6. Error and Precision in Measurement
7. Conclusion
Systems of Measurement

In chapter 10 we will examine measurements as they relate to geometry. There are many different systems and types of measurement to consider.

Types of Measurements

There are many different types of measurements. The types we will consider include:

- Time
- Length
- Weight
- Volume
- Temperature
Measuring Time

Some of the first systems of measurements developed may have been meant to measure time.

**Ancient Measures of Time**

- **Sumerian**
  The Sumerians had only two division points in the day. Midday, when the sun was overhead, and sunset.

- **Babylonians**
  Babylonians divided the day into 12 equal “hours” and then night. However, the lengths of these hours differed according to the season. They also divided the year into 360 days. (why?)

- **Greeks**
  The ancient Greeks divided the day into 24 hours with standard length.
Accuracy and Time Measurement

Not only the measures of time, but the technology used has become more useful and accurate over time.

Technology and Time Measurement

- **Sundials**
  Sundials in the form of obelisk date from as early as 3500 B.C. and were developed by many cultures including the Egyptians, Chinese, Greeks, and Romans. Most sundials were worked only in a given location.

- **Water Clock**
  Next to sundials, water clocks are possibly the oldest time-measuring device. Water clocks were thought to have existed in India and China as early as 3000 B.C. While the principles of the water clock are still used today, the use of water flow to power a clock is rare and more decorative than for functional accuracy.

- **Hourglass**
  The earliest evidence of the use of an hourglass is from the 14th century. Hourglasses were the first dependable, reusable, and reasonably accurate measure of time. They are functional at sea and not liable to freeze or affected by the weather.

- **Mechanical Clocks**
  While no clocks survive from medieval Europe, mentions in church records reveal that between 1280 and 1320 a new type of clock mechanism using weights and some oscillating mechanism was developed. This marks the beginning of the true mechanical clock.

- **Atomic Clock**
  The atomic clock uses an atomic resonance frequency to keep extremely accurate time. These clocks maintain a continuous and stable time scale. The first atomic clock was built in 1949 by the US National Bureau of Standards. Currently, one second is defined to be the duration of 9,192,631,770 periods of a cesium-133 atom.
Ancient Measures of Length

Ancient units of length were often based on lengths from the body.

### Ancient Units of Length

The following ancient units of length were based on various measurements taken from the body.

- **Cubit**
  The cubit is defined to be the distance from the tip of one’s elbow to the tip of the middle finger.

- **Foot**
  The foot was defined by the ancient Romans to be the actual length of Hercules’ Foot.

- **Yard**
  A yard is the distance from the tip of British king Edgar I nose to the tip of his middle finger on an outstretched arm.

- **Span**
  A span is the width of an open hand

- **Hand**
  A hand is the width of a closed hand

- **Fathom**
  The width of an outstretched arm is a fathom.
Roman Measurements of Length

The Romans under Alexander the Great developed a cohesive system of measurements of length.

- The foot was divided into 12 unciae.
- Five feet made a pace.
- A mile was 1000 paces.

Historical Note

Alexander the Great employed professional pacers to measure the distance his army traveled.
Modern Measures of Length - English System

There are two widely used systems of measurement for length employed today.

**English Measures of Length**

The English system of measurements for length is based on the inch.

- 1 inch (in) is the basic unit of measurement
- 1 foot (ft) is 12 inches
- 1 yard (yd) is three feet
- 1 mile (mi) is 1760 yards, or 5280 feet

**Example**

Convert 14 feet to inches and 48 inches to yards.
Modern Measures of Length - Metric System

The metric system of measurement was first proposed in 1670. It was adopted by the French in 1789 to help standardize measurements after the revolution and prevent the rich from cheating the poor.

The Meter

The basic unit of measurement in the metric system is the meter (m). It is defined to be one ten-millionth the distance from the line from the equator to the north pole passing through Barcelona, Paris, and Dunkirk.

Metric Prefixes

Other metric measures of length are powers of ten times a meter.

\[
\begin{align*}
milli- & \quad \frac{1}{1000} \\
centi- & \quad \frac{1}{100} \\
deci- & \quad \frac{1}{10} \\
deca- & \quad 10 \\
hecto- & \quad 100 \\
kilo- & \quad 1000
\end{align*}
\]

Conversions Between Systems

To convert between systems, use 1 inch \(\approx\) 2.5 centimeters.
Ancient Measures of Mass

Ancient measures of mass evolved from the Babylonian system in which the smallest unit was the weight of a carob plant seed (called a carot and later a grain).

Apothecaries’ System

The apothecaries’ system is an antiquated system of measurement used by apothecaries in England.

<table>
<thead>
<tr>
<th>Units</th>
<th>Grains</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grain</td>
<td>1</td>
</tr>
<tr>
<td>Scruple</td>
<td>20</td>
</tr>
<tr>
<td>Dram</td>
<td>60 (3 scruples)</td>
</tr>
<tr>
<td>Ounce</td>
<td>480 (8 drams)</td>
</tr>
<tr>
<td>Pound</td>
<td>5760 (12 ounces)</td>
</tr>
</tbody>
</table>

Troy System

The troy weight system is still used today for measuring the weight of precious metals, black powder, and gemstones. The units include:

<table>
<thead>
<tr>
<th>Unit</th>
<th>Grains</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grain</td>
<td>1</td>
</tr>
<tr>
<td>Pennyweight</td>
<td>24</td>
</tr>
<tr>
<td>Ounce</td>
<td>480 (20 pennyweights)</td>
</tr>
<tr>
<td>Pound</td>
<td>5760 (12 ounces)</td>
</tr>
</tbody>
</table>
Contemporary Mass Measurement Systems

The most common English system of pass used today is shown below.

Avoirdupois System (US customary)

<table>
<thead>
<tr>
<th>Unit</th>
<th>Pounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dram</td>
<td>(\frac{1}{256})</td>
</tr>
<tr>
<td>Ounce</td>
<td>(\frac{1}{16})</td>
</tr>
<tr>
<td>Pound</td>
<td>1</td>
</tr>
<tr>
<td>Quarter</td>
<td>25</td>
</tr>
<tr>
<td>Hundredweight</td>
<td>100</td>
</tr>
<tr>
<td>Ton</td>
<td>2000</td>
</tr>
</tbody>
</table>

Metric Measures of Mass

The metric system for measuring mass is based on the kilogram which is the mass equal to the international prototype of the kilogram. It is approximately 2.205 avoirdupois pounds.
Dry Measures of Volume

Measures of volume are divided into two groups: wet and dry. Dry measures are used for bulk commodities which are not liquid.

Imperial Measures of Dry Volume

Many units in this system are associated with particular goods: hogshead, barrel, bushel, peck, cord, bale, etc.

Metric Units of Dry Volume

In the metric system the basic units of dry volume are the liter and the cubic meter.

Conversions

- 1 pint ≈ 550.6 milliliters
- 1 quart = 2 pints ≈ 1.101 liters
- 1 gallon = 4 quarts ≈ 4.404 liters
- 1 peck = 8 quarts = 2 gallons ≈ 8.81 liters
- 1 bushel = 4 pecks ≈ 35.239 liters
Liquid Measures of Volume

Liquid volume has a different set of measurements, some of which overlap with dry volume.

**Imperial Units if Liquid Volume**

The basic measure of liquid volume in the imperial system is the teaspoon.

- Tablespoon (Tbsp) = 3 teaspoons (tsp)
- Liquid Ounce (oz) = 2 tablespoons
- Cup = 8 liquid ounces
- Pint = 2 cups
- Quart = 2 pints
- Gallon = 4 quarts

**Metric Units of Liquid Volume**

The basic measure of liquid volume in the metric system is the liter. The liter is defined to be the volume of a 10 centimeter cube (1000 cubic centimeters). It is approximately 34 liquid ounces.
Measures of Temperature

Again there are several systems for measuring temperature.

Fahrenheit

In 1714 Gabral Farenheight, the inventor of the mercury thermometer, came up with a scale for measuring temperature. One story claims that the lowest temperature he could measure outdoors became his 0 and his body temperature became 100. In this system water boils and $212^\circ \text{F}$ and freezes at $32^\circ \text{F}$.

Celsius

The Celsius temperature scale (previously called the centigrade scale) as developed by Anders Celsius in 1742. This system is commonly thought to be based on a boiling point of $100^\circ \text{C}$ and freezing point of $0^\circ \text{C}$ for water under one atmosphere of pressure. While these values are close, the true scale is based on absolute zero and the triple point of specially prepared water.
The division of a full circle into 360° is most likely linked to the Babylonian system of time in which a year had 360 days.

**Metric Angles**

When the metric system was first proposed, a measurement for angles was also suggested. In that system, there were 10 hours in a day, 10 days in a week, 10 weeks in a month, and 10 months in a year.

A circle would have 400° so that a right angle would measure 100°.

This system was actually implemented after the French revolution. It was eventually abolished by Napoleon as it was too confusing and did not conform to the systems in use elsewhere.
Error and Precision

As you have probably seen in your lab, any attempt to measure something must recognize the fact that approximation is involved.

**Precision**

The precision of a measurement is a measure of how close our readings are to the true amount. This is based on the accuracy of our measuring device.

**Error**

Error in a measurement is caused not by the accuracy of a measuring device, but by mistakes made in the measuring or recording process.
**Error vs. Precision Examples**

**Example**
Suppose you claim a log is 13 feet long. Your measurement is accurate if the log is 13.2 feet long and your precision is only one foot. However, with a one foot precision, your measurement would be in error if the log were 13.8 feet long.

**Measuring Error**
One method of measuring error is to calculate the greatest possible error. Another is the relative error (ratio of actual error in the measurement to the actual measured amount).

**Example**
In the example above, if the log were 13.8 feet long, the GPE is 0.5 feet.

**Example**
The relative error would be \( \frac{0.5}{13} = 3.8\% \).
Important Concepts

Things to Remember from Section 10.1

1. Units of Measurement for:
   - Time
   - Length
   - Weight/Mass
   - Volume
   - Temperature

2. Converting between Metric and Imperial measures.

3. Measuring error