Outline

1. Introduction to Integers
2. Representing Integers
3. Modeling Integer Operations
   - Integer Addition
   - Integer Subtraction
   - Integer Multiplication
   - Integer Division
4. Conclusion
Known Number Sets

Last quarter we spent much of our time with two simple sets of numbers.

Natural Numbers
The natural numbers are the counting numbers as shown below.

\[ \mathbb{N} = \{1, 2, 3, 4, \ldots \} \]

Whole Numbers
The whole numbers add zero to the natural numbers.

\[ \mathbb{W} = \{0, 1, 2, 3, 4, \ldots \} \]

During the first few weeks of this quarter, we will extend these sets of numbers to include integer, rational, and real numbers.
The first new set of numbers we will consider is the integers.

**Integers**

The integers include positive and negative natural numbers and zero.

\[ \mathbb{Z} = \{ \ldots, -2, -1, 0, 1, 2, \ldots \} \]

We can use a Venn Diagram to show the relationship between natural, whole, and integer numbers.
Why Use Integers

Is the concept of a negative number really that natural or useful?

Example
Suppose I owe you $2. One might say that I have \(-$2\). Is this natural?

Example
Located in California, Death Valley is 282 feet below sea level. Its elevation is \(-282\) feet.

Example
As a cold front moves in, the temperature drops by 2 degrees per hour. The rate of change in temperature is \(-2^\circ/\text{hour}\).
The Set Model

When we started working with natural and whole numbers, there were several different models which were useful for representing the concept of a number.

**The Set Model**

In the set model, a natural number is represented as a set of objects. For example, we might represent the number 3 as a pile of 3 cookies.

**Example**

Describe how one could expand the set model to represent all integers, including negative numbers. Draw a picture to represent the integers 0 and -2.
The Number Line Model

Another method which we used to represent natural numbers is the concept of a number line.

**Number Lines**

With whole numbers, a number line starts at 0 and extends infinitely to the right. The natural numbers appear at set distances along that number line as it moves to the right.

**Example**

Describe and draw a picture of how one could expand the number line described above to represent all integers. Use a number line to represent -2.
Using the Set Model

While your book divides integer addition into four cases (+/+, +/−, −/+), we will examine them all together using the set model.

Example

Use the idea of “cookies” and “anti-cookies” to model the following addition problems.

1. 3 + 5
2. 3 + (−2)
3. −7 + 4
4. −3 + (−4)

What are some of the advantages and disadvantages of the set model for integer addition?
If the concept of an “anti-cookie” seems unnatural to you, the number line model for integer addition may make more sense.

**Example**

Use a number line to model the following addition problems.

1. $3 + 5$
2. $3 + (-2)$
3. $-7 + 4$
4. $-3 + (-4)$

What are some of the advantages and disadvantages of the number line model for integer addition?
While we did not need to consider the four cases mentioned previously to model integer addition, it is interesting to consider the results of each of these types of integer addition problems.

### Results of Integer Addition

Suppose that $a$ and $b$ are both integers with $a < b$. Then there are four possibilities for the sum $a + b$ depending on the sign of $a$ and $b$:

<table>
<thead>
<tr>
<th>Case</th>
<th>$a$</th>
<th>$b$</th>
<th>$a + b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$+$</td>
<td>$+$</td>
<td>larger than $a$ or $b$ and $+$</td>
</tr>
<tr>
<td>II</td>
<td>$+$</td>
<td>$-$</td>
<td>smaller than $a$ but $+$</td>
</tr>
<tr>
<td>III</td>
<td>$-$</td>
<td>$+$</td>
<td>larger than $a$ and $-$</td>
</tr>
<tr>
<td>IV</td>
<td>$-$</td>
<td>$-$</td>
<td>smaller than $a$ or $b$ and $-$</td>
</tr>
</tbody>
</table>

### Example

What happens in each of the cases above if $a = b$?
The number line model and the cases seen above tie in nicely with the concept of the absolute value of an integer.

**Absolute Value**

The absolute value of an integer $a$, written $|a|$, is the distance from zero to the number $a$ on a number line.

**Example**

Use a number line to find $|5|$ and $|-5|$.

Is there a quicker way to find an absolute value than drawing out a number line and finding distances?
Using the Set Model

Since subtraction is the opposite of addition, it makes sense that we should be able to use similar methods to model integer subtraction.

Example

Use the idea of cookies and “anti-cookies” to model the following subtraction problems.

1. 4 - 2
2. 2 - 4
3. 7 - (-2)
4. -3 - 2

Do you think that this model works as well for subtraction as it did for addition? Explain.
Using the Number Line Model

The number line model can also be adapted to subtraction simply by remembering that if we subtract a number, we move left on the number line.

Example

Use a number line to model the following subtraction problems.

1. 4 - 2
2. 2 - 4
3. 7 - (-2)
4. -3 - 2

Do you think that the number line model or the set model is easier to understand when subtracting? Explain.
A General Rule Explained

Models such as the two we have just seen can be useful for explaining why a general rule works. Consider the following rule of addition and subtraction.

**Equivalence of Addition and Subtraction**

If $a$ and $b$ are any integers, then:

$$a - (-b) = a + b$$

and also

$$a - b = a + (-b)$$

**Example**

Use a number line to justify the two equations given above.
Multiplication as Repeated Addition

Integer multiplication can be more difficult than addition or subtraction. One method of modeling integer multiplication is by using repeated addition.

Example

Use repeated addition to find and justify each of the following products.

1. $3 \times 4$
2. $3 \times -4$
3. $-3 \times 4$
4. $-3 \times -4$

What other methods of modeling integer multiplication did you come up with in your lab?
There are several ways in which we can model integer division.

**Example**

Use a set model (also called the partition model) to find \(-12 \div 4\).

**Example**

Use a number line to model \(4 \div -2\).

**Example**

How could you model \(-6 \div -2\)? Come up with at least two different methods.
Important Concepts

Things to Remember from Section 5.1

1. The definition of the set of Integers and how they relate to other number sets.

2. Methods for modeling integers

3. Methods for modeling operations on integers

4. The definition of an absolute value