MATH 113
Section 8.2: Two-Dimensional Figures

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Outline

1. Classifying Two-Dimensional Shapes

2. Polygons
   - Triangles
   - Quadrilaterals

3. Other Two Dimensional Figures

4. Conclusion
One of the important parts of geometry is classifying shapes and learning their properties. We begin our study of two dimensional figures with just such an exercise.

Example

How would you classify these shapes? List several different ways.
Figures and Definitions

To help us find standard classifications for shapes, we start with a few definitions.

**Simple Closed Curves**

A simple closed curve is a curve which we can trace without going over a point more than once while beginning and ending at the same point.

**Example**

Which of the previous figures are simple closed curves?

**Questions**

What properties of a curve are being described by the terms:

- Simple
- Closed
More Definitions

Let’s examine several of these terms in more detail.

**Examining Each Term**

- **Curve** - a straight or “curvy” set of connected points
- **Closed** - starting and ending at the same point
- **Simple** - does not cross itself

**Example**

Given these specifically defined terms, draw each of the following.

1. a simple closed curve
2. a simple open curve
3. a non-simple closed curve
4. a non-simple open curve
The Jordan Curve Theorem

Before we start talking about specific types of figures, we will look at one important general theorem in geometry.

**Jordan Curve Theorem**

Let $c$ be a simple closed curve in the plane. Then the complement of the image of $c$ consists of two distinct connected components. One of these components is bounded (the interior) and the other is unbounded (the exterior).

**History of the Theorem**

While this may seem intuitively clear, it is not easy to show.

- First attempt by Bernard Bolzano in (1781-1848)
- Then by Camille Jordan (1838-1922)
- Finally proved in 1905 by Oswald Veblen
- Rigorous formal proof of over 200,000 lines produced in 2005.
We start our detailed exploration of two-dimensional figures with the polygon.

### The Polygon
A polygon is a simple closed curve composed only of line segments. The line segments are called sides and the points where they meet are the vertices.

### Classification of Polygons
Polygons can be classified by the number of sides.

- triangle - 3 sides
- quadrilateral - 4 sides
- pentagon - 5 sides
- hexagon - 6 sides
- heptagon - 7 sides
- octagon - 8 sides
- nonagon - 9 sides
- decagon - 10 sides
Classifying Polygons

There are several general ways to classify polygons.

**Convex Polygons**

A convex polygon is one in which a line segment connecting any two points on the polygon lies completely inside the polygon.

**Concave Polygons**

In a concave polygon we can draw a line segment connecting two points on the polygon which lies in the exterior of the polygon.

**Regular Polygons**

A polygon in which all sides have the same length and all interior angles have the same measure is called regular.

**Diagonals**

A diagonal is a line segment which joins two non-adjacent vertices.
The triangle with the fewest sides is of particular importance.

A triangle is a polygon with exactly three sides.

One of the important properties of triangles is that they are a stable, rigid structure.

Triangles can be classified in several different ways.

- By sides - equilateral, isosceles, scalene.
- By angles - right, obtuse, acute

Example

Draw a Venn Diagram to show the interaction between right and isosceles triangles.
The Median and Centroid

In the next few slides we will look at several types of line segments and the point at which they all intersect. The first of these is discussed below.

### The Median

The median of a triangle is the line segment that connects a vertex to the midpoint of the opposite side of the triangle.

### The Centroid

As there are three vertices in a triangle, there are three medians. These three line segments will always intersect at a single point, called the centroid (center of gravity) of the triangle.

### Example

Draw a triangle and find its centroid.
Perpendicular Bisectors and the Circumcenter

The next set of three line segments and their central point of intersection is made up of perpendicular bisectors.

**Perpendicular Bisector**

A perpendicular bisector is a line segment passing through the midpoint of a side which is perpendicular to that side.

**Circumcenter**

As there are three sides in a triangle, there are three perpendicular bisectors. These three line segments will always intersect at a single point called the circumcenter. The circumcenter is equidistant from the three vertices of the triangle.

**Example**

Draw a triangle and find its circumcenter.
Angle Bisectors and Incenters

Another center of a triangle can be located by finding the intersection of the angle bisectors.

**Angle Bisectors**

An angle bisector is a line segment which bisects (divides in two) an internal angle of a triangle.

**The Incenter**

As there are three angles in a triangle, there are three angle bisectors. These three line segments will always intersect at a single point called the incenter of the triangle. This is the point equidistant from all three sides.

**Example**

Draw a triangle and find its incenter.
The Altitudes and Orthocenter

The final set of line segments and the center they define is made up of altitudes.

Altitude
An altitude of a triangle is a line segment perpendicular to a side of the triangle and connected to the opposite vertex.

The Orthocenter
As there are three sides in a triangle, there are three altitudes. These three line segments will always intersect at a single point called the incenter of the triangle.

Relationships
The centroid, orthocenter, and circumcenter are all colinear. They are the same point only if the triangle is equilateral.
Triangles and Congruence

Two polygons are congruent if all corresponding parts of the figure are congruent. That means corresponding sides have the same length, and corresponding angles have the same measure.

Example

Suppose two triangles, \( \triangle ABC \) and \( \triangle XYZ \), are congruent. What does this mean about the line segments and angle measures of these triangles?

Congruence in Triangles

In your lab we will see several methods for showing that two triangles are congruent. They include:

- Side-Angle-Side
- Angle-Side-Angle
- Side-Side-Side
As with triangles, there are several ways to classify quadrilaterals, depending on angles and side lengths.

### Classifying Quadrilaterals

The following are several types of quadrilaterals.

- **trapezoid** - at least one pair of parallel sides
- **parallelogram** - both pairs of opposite sides are parallel
- **kite** - two pairs of adjacent sides are congruent
- **rhombus** - all sides are congruent
- **rectangle** - all angles are congruent
- **square** - all four sides and all four angles are congruent.

### Example

Draw a figure showing the interaction between types of quadrilaterals.
There are many types of curved figures, but perhaps the most familiar is the circle.

A circle is the set of points in a plane which are equidistant from a given fixed point, called the center.

### Naming Parts of a Circle

- $C$ is the center
- $\overline{CA}$ is a radius
- $\overline{AB}$ is a diameter
- $\overline{XY}$ is a chord
- $\widehat{AD}$ is an arc
- $\overrightarrow{PQ}$ is a tangent line
Geometry can also be done using a coordinate system and equations.

**The Cartesian Coordinate System**

The Cartesian coordinate system consists of two number lines perpendicular to each other. The point at which they meet is called the origin.

**Distance**

The distance between any two points \((x_1, y_1)\) and \((x_2, y_2)\) is:

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]
By giving the coordinates of vertices, we can describe figures using coordinate geometry.

**Example**

What type of figure is the quadrilateral $ABCD$ where $A = (0, 0)$, $B = (1, 1)$, $C = (2, 1)$, and $D = (0, 1)$?

We claim that $ABCD$ is a parallelogram and a kite.

- Find the length of each side
- Find the slope of each line
Important Concepts

Things to Remember from Section 8.2

1. Classification of curves as simple and closed
2. Names of polygons by number of sides (up to 10-sided figures)
3. Classification and parts of triangles
4. Classification of quadrilaterals
5. Naming parts of a circle
6. Basic Coordinate Geometry