Exam III Review Sheet  
MATH 121B, Winter 2009

This exam will cover sections 2.6-4.1 in your text. You should know general terms and definitions from each of these sections, review the homework and quizzes given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Setting-up and solving problems involving variation (direct, inverse, joint, and combined).
2. Writing a quadratic function \( f(x) = ax^2 + bx + c \) in the form \( f(x) = a(x - h)^2 + k \).
3. Sketching graphs of quadratic functions by finding the vertex, \( x \)-intercepts, and opening direction.
4. Finding the minimum or maximum value of a quadratic function.
5. Using synthetic division to evaluate a polynomial or divide by a linear factor.
6. Using the rational roots theorem to locate potential zeros of a polynomial function.
7. Identifying the multiplicity of a zero.
9. Solving polynomial equations involving square roots, quadratic type expressions, and absolute values.
10. Graphing polynomial functions using symmetry, \( x \)-intercepts, long term behavior, degree, etc.
11. Solving polynomial or rational expression inequalities.
12. Graphing rational expressions and locating vertical, horizontal, and slant asymptotes.

Below is a list of sample problems.

1. Set-up and solve the following problems involving variation.
   (a) If \( v \) varies directly as \( t \) and \( v = 3 \) when \( t = 1 \), for what value of \( t \) will \( v = 9 \)?
   (b) If \( x \) varies jointly as \( y \) and \( z \) and \( z = 4 \) when \( x = 3 \) and \( y = 1 \), find \( x \) when \( y = 2 \) and \( z = 1 \).
   (c) The distance that a ball will travel is directly proportional to the speed at which it is thrown and inversely proportional to the headwind. If a ball thrown at 30 feet per second in a headwind of 5 feet per second will travel 50 feet, how far will a ball thrown at 35 feet per second into a 3 foot-per-second headwind travel?

2. Sketch the graph of each of the following functions by writing it in \( a(x - k)^2 + k \) form. Locate the vertex, \( x \)-intercepts, and indicate the minimum or maximum value of each function.
   (a) \( f(x) = x^2 - 5x + 1 \)
   (b) \( h(x) = -2x^2 + 6x + 9 \)
   (c) \( g(x) = \frac{1}{3}x^2 + 4x - 1 \)
   (d) \( P(x) = -x^2 + 2x - 6 \)

3. Perform the indicated task with each polynomial.
   (a) Find \( \frac{x^4-x^3+x^2-x+2}{x^2-2} \) using synthetic division.
   (b) Give the quotient and remainder when \( 3x^4 - 5x^3 - 20x - 5 \) divided by \( x^2 + x + 3 \).
   (c) Use synthetic division to find the remainder when \( x^{101} - 1 \) is divided by \( x - 1 \).
   (d) Find \( P\left(\frac{1}{4}\right) \) using synthetic division is \( P(x) = x^3 - x + 1 \).
4. Use the rational roots theorem and synthetic division to factor each polynomial completely.
   (a) \( P(x) = x^4 + x^3 + 7x^2 + 9x - 18 \)
   (b) \( P(x) = x^3 - 5x + 3 \)
   (c) \( Q(x) = 2x^4 + 5x^3 + 3x^2 + 15x - 9 \)

5. Answer each question involving polynomials and zeros.
   (a) If \( 3 - i \) is a root of \( P(x) \), what is another root?
   (b) Factor \( P(x) = 4x^3 - x^2 + x - 4 \) completely if \( 2i \) is a root.
   (c) Find the best integer upper bound on the positive real roots of \( x^4 - x^3 - 9x^2 + 9x + 4 \).
   (d) Find the best integer lower bound on the negative real roots of \( x^5 - x^4 - 2x^2 - 4x - 5 \).
   (e) Determine the possible number of positive and negative real roots in the polynomials above.

6. Solve each equation.
   (a) \( \sqrt{n} + 4 + \sqrt{n - 1} = 5 \)
   (b) \( x^{\frac{2}{3}} + 10 = 7x^{\frac{1}{3}} \)
   (c) \( |x^2 - 2x| = |3x - 6| \)
   (d) \( (2y + 1)^{\frac{2}{3}} = 3 \)

7. Graph the following by finding \( x \)-intercept behavior, long-term behavior, the degree, and any other helpful information.
   (a) \( P(x) = x^4 - 2x^3 - 8x + 16 \)
   (b) \( P(x) = x^4 - 9x^2 \)
   (c) \( P(x) = 2x^3 - x^2 - 18x + 9 \)

8. Graph the following rational functions by finding vertical and horizontal asymptotes, \( x \)-intercepts, and other helpful information.
   (a) \( \frac{x+1}{x-3} \)
   (b) \( \frac{x-2}{x+1} \)
   (c) \( \frac{x^2-1}{x^2+7x+10} \)

9. Solve each of the following inequalities by constructing a sign graph.
   (a) \( 8x^2 + 1 < 6x \)
   (b) \( 4x^3 - 400x^2 - x + 100 \geq 0 \)
   (c) \( \frac{x+10}{x+2} < 5 \)
   (d) \( \frac{x^2-3x+2}{x^2-7x+12} \geq 0 \)

10. Perform each task involving exponential functions.
    (a) Let \( f(x) = 2^{1-x} \). Find \( f(1) \) and \( f(-1) \).
    (b) Sketch a graph of \( f(x) = 3^x \).
    (c) Use transformations to sketch a graph of \( g(x) = 2 \cdot 3^{x+1} - 1 \).
    (d) Solve the equation \( -3^x = -27 \).