1. Use mathematical induction to show that the following statements are true for all positive integers $n$.

(a) $9^n - 1$ is divisible by 4

**Base Case:** $n = 1$, $9^1 - 1 = 9 - 1 = 8$ is divisible by 4.

**Inductive Step:** Assume $P_k : 9^k - 1 = 4r$ and prove $P_{k+1} : 9^{k+1} - 1$ is divisible by 4. Starting with $9^k - 1 = 4r$ multiply both sides by 9 to get $9(9^k - 1) = 9(4r)$ so that $9^{k+1} - 9 = 4(9r)$ and adding 8 to both sides, $9^{k+1} - 1 = 4(9r) + 8$. Therefore, $9^{k+1} - 1$ is divisible by 4, and thus, by induction, $9^n - 1$ is divisible by 4 for all positive integers $n$.

(b) $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

**Base Case:** $n = 1$, $1 = \frac{1(1+1)}{2} = 1$

**Inductive Step:** Assume $P_k : 1 + 2 + \ldots + k = \frac{k(k+1)}{2}$ and prove $P_{k+1} : 1 + 2 + \ldots + (k+1) = \frac{(k+1)(k+2)}{2}$. Starting with $P_k$, add $k + 1$ to both sides to get $1 + 2 + \ldots + k + (k + 1) = \frac{k(k+1)}{2} + (k + 1)$. Now, the left-hand side patches $P_{k+1}$, so we must work with the right-hand side. $\frac{k(k+1)}{2} + (k + 1) = \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$. Therefore, by induction, the formula holds for all positive integers.

(c) $\frac{a^n}{a} = a^{n-1}$

**Base Case:** $n = 1$, $\frac{a^1}{a} = a = 1 = a^0 = a^{1-1}$

**Inductive Step:** Assume $P_k : \frac{a^k}{a} = a^{k-1}$ and show $P_{k+1} : \frac{a^{k+1}}{a} = a^{k+1-1}$. Starting with $P_k$, multiply both sides by $a$ to get $\frac{a^k}{a}a = a^{k-1}a$ so that $\frac{a^k}{a}a = a^k$ and hence $\frac{a^{k+1}}{a} = a^{k+1-1}$. Therefore, by induction, the formula holds for all positive integers.

2. If $a_1, a_2, \ldots, a_n, \ldots$ is an arithmetic sequence, find each of the following.

(a) $a_2 = 2 + 5 = 7$, $a_{10} = 2 + (9)5 = 47$, and $a_{50} = 2 + (49)5 = 247$.

(b) $a_{12} = -9 + 11d = -31$ so $d = -2$. Thus, $a_{45} = -9 + 44(-2) = -97$.

(c) $a_2 = 40 + d$ so $d = 55$ and $S_{15} = \frac{15}{2}(2(40) + 14(55)) = 6375$.

(d) $a_6 = a_1 + 5d$ and $a_{10} = a_1 + 9d$ so $a_1 = -4, d = 6$ thus $S_{10} = \frac{10}{2}(-4 + 50) = 230$.

3. If $a_1, a_2, \ldots, a_n, \ldots$ is a geometric sequence, find each of the following.

(a) $a_5 = 81 \left(\frac{1}{3}\right)^4 = 1$, $a_{10} = 81 \left(\frac{1}{3}\right)^9 = \frac{1}{243}$.

(b) $a_1 = 2187$, $r = 1$ thus $S_{10} = 10(2187) = 21870$.

(c) $S_{10} = 20 \left(\frac{12-2^{10}}{1-2}\right) = 20(1023) = 20460$.

4. Evaluate each of the following.

(a) $362880$

(b) $30! = 264 \times 17100720$

(c) $\frac{13!}{5!} = \frac{13!}{5!} = 13! 10! \cdot 9! 8! \cdot 7! 6! 5! 4! 3! 2! 1! = 1287$

(d) $\frac{12!}{3!} = \frac{12!}{3!} = \frac{12!}{3!} = \frac{12!}{3!} = 220$

(e) $\frac{15!}{5! 10!} = \frac{15!}{5! 10!} = \frac{15!}{5! 10!} = \frac{15!}{5! 10!} = 3003$
5. Expand each power of a binomial, or find the term indicated.

(a) $x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$

(b) $8 + 36x + 54x^2 + 27x^3$

(c) 2nd: $-20a^4$ and 5th: $1280a$

(d) 5th: $5985u^{17}v^4$ and 18th: $5985u^4v^{17}$