Exam II Review Sheet
MATH 122, Winter 2008

This exam will cover sections 6.5-8.2 in your text. You should know general terms and definitions from each of these sections, review the homework and quizzes given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Using identities and algebra to solve trigonometric equations.
2. Using the law of sines to solve ASA, AAS, and SSA triangles.
3. Using the law of cosines to solve SSS and SAS triangles.
4. Vector arithmetic: magnitude, addition, scalar multiplication, dot-product.
5. Setting up and solving application problems involving vectors.
6. Converting complex numbers between $a + bi$ and trigonometric forms.
7. Using the trigonometric form of complex numbers to multiply and/or divide.
8. Finding all $n$th roots of a complex number.
9. Plotting polar points and graphing polar equations.
10. Converting between polar and rectangular points or equations.
11. Solving systems of equations graphically.
12. Solving systems of equations by substitution.
13. Solving systems of equations by addition.

A sampling of problems representative of the types you will see on the exam is provided below.

1. Find exact solutions for real $x$ on the interval $[0, 2\pi)$ and $\alpha$ in degrees on the interval $[0^\circ, 360^\circ)$
   (a) $\sqrt{3}\tan x + 1 = 0$
   (b) $\sqrt{2}\csc x = 2$
   (c) $2\sin^2\alpha = -\sin 2\alpha$

2. Find all solutions for real $x$ and $\alpha$ in degrees using a calculator and four decimal places.
   (a) $5\cos x - 2 = 0$
   (b) $1.3224\sin \alpha + 0.4732 = 0$
   (c) $2\sin x = \cos 2x$
   (d) $\cos 2\alpha + 10\cos \alpha = 5$

3. Solve each triangle according the the labeled diagram and the information given.

4. Given the vectors $\vec{u} = \langle 2, -3 \rangle$, $\vec{v} = \langle -5, -4 \rangle$, and $\vec{w} = \langle 0, 7 \rangle$, find the following.
   (a) $2\vec{u} - 3\vec{w}$
   (b) $5(\vec{v} + \vec{u})$

\begin{align*}
\text{(a)} & \quad \alpha = 52^\circ, \gamma = 105^\circ, c = 47 \text{ meters} \\
\text{(b)} & \quad \beta = 43^\circ, \gamma = 36^\circ, a = 92 \text{ cm.} \\
\text{(c)} & \quad \alpha = 38.9^\circ, a = 30 \text{ feet, } b = 42.7 \text{ feet} \\
\text{(d)} & \quad \alpha = 120^\circ20', b = 5.73 \text{ km.}, c = 10.2 \text{ km.} \\
\text{(e)} & \quad a = 6.9 \text{ in.}, b = 31.3 \text{ in.}, c = 23.4 \text{ in.} \\
\text{(f)} & \quad \gamma = 46.7^\circ, a = 18.1 \text{ yards}, b = 22.6 \text{ yards}
\end{align*}
5. Set-up and solve the following problems using geometric vectors.
   (a) A cruise ship makes 30 knots in still waters. If a current of 3.5 knots flows from the west, what bearing should the ship take in order to travel due north relative to the ocean floor?
   (b) In an attempt to remove a stump from your front yard, two of your friends have tied chains from the stump to their trucks, and attempt to pull at the same time. If your first friend’s truck pulls with 800 pounds of force due north and your second friend’s truck pulls with 650 pounds of force 42° east of north, what is the compass direction and magnitude of the resultant force?

6. Plot each point in the complex plane, and convert between $a + bi$ and trigonometric forms.
   (a) $5 \left[ \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right]$  
   (b) $3 + 5i$  
   (c) $-3i$  
   (d) $7 \left[ \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right]$

7. Answer each question about complex numbers by converting to trigonometric form to simplify computation.
   (a) Find all cube roots of $i$, writing your answers in rectangular form.
   (b) If $z_1 = 8 \cos 25^\circ + i \sin 25^\circ$ and $z_2 = 4 \cos 20^\circ + i \sin 20^\circ$, find $z_1 z_2$ and $\frac{z_1}{z_2}$.
   (c) Find $(1 + i\sqrt{3})^4$ in $a + bi$ form.
   (d) Show that $4 \cos 15^\circ + i \sin 15^\circ$ is the square root of $8\sqrt{3} + 8i$.
   (e) Solve the equation $x^3 - 27 = 0$.

8. Convert each equation from polar to rectangular or rectangular to polar form and graph.
   (a) $r = 10 \cos \theta$  
   (b) $x^2 + y^2 = 6y$  
   (c) $r = 6$  
   (d) $\theta = \frac{\pi}{3}$

9. Solve the following systems of equations by graphing.
   
   3x - 2y = 8  
   x + 3y = -1  
   2x + y = 7  
   3x - 2y = 0

10. Solve the following systems of equations using either the substitution or addition methods. Use each method at least once.
    
    4x - 3y = -8  
    -2x + \frac{3}{2}y = 4  
    x_1 + 2x_2 + x_3 = 1  
    2x_1 + 3x_2 + 4x_3 = 3  
    x_1 + 2x_2 + x_3 = 3
    
    x_1 + 2x_2 - x_3 = 2  
    2x_1 + 3x_2 + x_3 = -3  
    x_1 - 2x_2 = 1  
    2x_1 - x_2 = 0
    
    x_1 + 5x_2 = -1  
    x_1 - 3x_2 = -2

11. A piggy bank contains 30 coins worth $1.90.
    (a) If the bank contains only nickels and dimes, how many coins of each type does it contain?
    (b) If the bank contains nickels, dimes, and quarters, how many coins of each type does it contain?