Analytic Geometry and Calculus I (MATH 181)
Final Exam Review Sheet, Spring 2008

This exam will cover sections 1.1-4.4 in your text. You should know general terms and definitions from each of these sections, review previous exams, homework assignments, and review sheets, and pay particular attention to the subjects and practice problems mentioned below.

1. Chapter 1
   (a) Stating and using the \( \epsilon - \delta \) definition of a limit.
   (b) Evaluating limits by direct substitution or special trigonometric formulas.
   (c) Stating and using the definition of continuity.
   (d) Identifying and classifying points of discontinuity.
   (e) Evaluating one-sided limits and infinite limits.

2. Chapter 2
   (a) Stating and using the limit definition of a derivative.
   (b) Finding derivatives analytically.
   (c) Applying the chain rule to differentiate compound functions.
   (d) Implicit differentiation and related rate problems.

3. Chapter 3
   (a) Stating and applying the Mean Value Theorem.
   (b) Identifying intervals on which a function is increasing, decreasing, concave up, or concave down.
   (c) Finding relative extrema using critical numbers and the first or second derivative tests.
   (d) Optimization problems (minimization/maximization).
   (e) Applying Newton’s method to approximate real zeros of a function.
   (f) Finding differentials and using them to approximate function values or propagation of error.

4. Chapter 4
   (a) Finding anti-derivatives and indefinite integrals.
   (b) Solving basic differential equations with initial conditions.
   (c) Approximating area under a curve using upper and lower sums.
   (d) Using Riemann Sums to find the definite integral.
   (e) Stating and applying the Fundamental Theorem of Calculus to evaluate definite integrals.
   (f) Finding the average value of a function on an interval
   (g) Using the second fundamental theorem of calculus.

Below is a sampling of problems representative of the types you will see from chapter 4. See previous review sheets and exams for problems from chapters 1-3.

1. Approximate the area of the region bounded between the graph of the given function and the \( x \)-axis over the indicated interval. Use both upper and lower sums with 5 subdivisions.
   (a) \( f(x) = 3x - 4 \) on \([2, 5]\]
   (b) \( f(x) = x^2 + 3 \) on \([0, 1]\]
   (c) \( f(x) = 2x - x^3 \) on \([0, 1]\)
2. Using the limiting process to find the exact area of each of the regions mentioned above.

3. Estimate the value of each integral using 4 subdivisions.
   (a) \( \int_{1}^{2} (x^2 + 1) \, dx \)
   (b) \( \int_{2}^{4} (x^3 + 4) \, dx \)
   (c) \( \int_{0}^{1/2} 4 \cos \pi x \, dx \)

4. Evaluate each definite integral using the Fundamental Theorem of Calculus.
   (a) \( \int_{2}^{5} (-3v + 4) \, dv \)
   (b) \( \int_{-1}^{1} (\sqrt{t} - 2) \, dt \)
   (c) \( \int_{1}^{4} (3 - |x - 3|) \, dx \)
   (d) \( \int_{\pi/4}^{\pi/2} (2 + \cos x) \, dx \)

5. Find the average value of the function \( f(x) = x - 2\sqrt{x} \) on the interval \([0, 2] \).

6. Differentiate each \( F \).
   (a) \( F(x) = \int_{0}^{x} t(t^2 + 1) \, dt \)
   (b) \( F(x) = \int_{0}^{x} t \cos t \, dt \)
   (c) \( F(x) = \int_{0}^{x} \sin \theta^2 \, d\theta \)
   (d) \( F(x) = \int_{x}^{x+2} (4t + 1) \, dt \)