Exam I Review Sheet  
MATH 181, Winter 2004

This exam will cover sections P.1-1.5 in your text. You should know general terms and definitions from each of these sections, review the homework given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Graphing lines and functions.
2. Finding the equation of a line.
3. Finding limits graphically, sketching graphs by hand or using a graphing calculator.
4. Stating and using the $\epsilon$-$\delta$ Definition of a Limit.
5. Evaluating limits analytically, by direct substitution or special trigonometric formula.
7. Stating and using the Definition of Continuity.
8. Identifying points of discontinuity, both removable and non-removable.
10. Working with one-sided limits.
12. Stating and using the Definition of Infinite Limits.

Below is a sampling of problems representative of the types you will see on the exam.

1. Use the $\epsilon$-$\delta$ definition of a limit to show that $\lim_{x \to 3} (6x - 2) = 16$.
2. Use the definition of a limit to show that $\lim_{x \to 1} (4x + 1) = 5$.
3. Use graph (i) of $f(x)$ to give a numerical estimate of a number $\delta > 0$ such that $|f(x) - 0.3| < 0.1$ whenever $0 < |x - 0.5| < \delta$. Justify your answer.

![Graph (i)](image1)

4. Use graph (ii) shown above to estimate the following values, if they exist.
   
   (a) $\lim_{x \to -2} f(x)$
   (b) $\lim_{x \to 2} f(x)$
   (c) $\lim_{x \to 5} f(x)$
   (d) $f(5)$
   (e) All values of $x$ at which $f(x)$ is discontinuous.
   (f) The type of discontinuity exhibited at the point(s).

![Graph (ii)](image2)
5. For each of the following limits, find the numerical value or state that the limit does not exist.

(a) \[ \lim_{x \to 2} \frac{x^2 - 7x + 10}{x - 2} \]

(b) \[ \lim_{x \to 0} \frac{\sin x}{3x} \]

(c) \[ \lim_{x \to 3} \frac{x^2 - 4x + 3}{x - 3} \]

(d) \[ \lim_{x \to 2} \sqrt{3x - 5} \]

(e) \[ \lim_{x \to 2} \frac{5}{3x - 6} \]

(f) \[ \lim_{x \to 3^+} \frac{1}{x - 3} \]

(g) \[ \lim_{x \to 0} \frac{\tan x}{x \cos x} \]

(h) \[ \lim_{x \to 3^+} \frac{|3 - x|}{3 - x} \]

6. Give the equations of any vertical asymptotes for the graph of the function \( f(x) = \frac{3}{x^2 - 1} \).

7. Suppose that \( \lim_{x \to 3} f(x) = 4 \) and suppose that \( \lim_{x \to 3} g(x) = 0 \). For each of the following, give a numerical value or state that the answer does not exist.

(a) \[ \lim_{x \to 3} (3f(x) + g(x)) \]

(b) \[ \lim_{x \to 3} \frac{f(x)}{g(x)} \]

(c) \[ \lim_{x \to 3} (f(x)g(x)) \]

8. For what values of \( x \) is each of the following functions continuous? Give a brief rational for your answers.

(a) \( f(x) = 6x^4 + 4x + 1 \)

(b) \( g(x) = \frac{x^2 - 9}{x - 3} \)

9. Let \( f(x) = x^7 + 2x - 1 \). Answer the following questions.

(a) Give the numerical values of \( f(0) \) and \( f(1) \).

(b) What do you know about a solution of the equation \( x^7 + 2x - 1 = 0 \)? Name any important theorems that you may which to invoke.

10. Suppose that \( f \) is a function. Answer the following questions in the case that (i) \( f \) is continuous at 2 and (ii) \( f \) is not necessarily continuous.

(a) If \( f(2) = 4 \) can anything be said about \( \lim_{x \to 2} f(x) \)? Why or why not?

(b) If \( \lim_{x \to 2} f(x) = 4 \) can anything be said about \( f(2) \)? Why or why not?

11. Let \( f(x) \) be defined as follows:

\[ f(x) = \begin{cases} 
  x^3 - 3x + 1 & \text{if } x < 2 \\
  2x + 1 & \text{if } x \geq 2 
\end{cases} \]

(a) Give the values of \( f(0) \) and \( f(5) \).

(b) For what values of \( x \) is \( f \) continuous, and for what values is it discontinuous? Give reasons for your assertions.

(c) Can you be sure that the equation \( f(x) = 0 \) has a solution? Give reasons for your answer, and name any important theorems you use.

12. Suppose that a function \( f(x) \) has the property that \( \frac{\sin x}{x} \leq f(x) \leq x^2 + 1 \) for all \( x \) in the interval \((-1,1)\). Find \( \lim_{x \to 0} f(x) \) and name any important theorems you use.