This exam will cover sections 2.3-3.3 in your text. You should know general terms and definitions from each of these sections, review the homework given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Applying various rules for differentiation including:
   (a) previously seen rules (constant, sum/difference, power rule).
   (b) the product rule.
   (c) the quotient rule.
   (d) the chain rule.
2. Proving the Product Rule and/or the Quotient Rule (p. 119, 121).
3. Using implicit differentiation on implicitly defined functions.
5. Stating the Extreme Value Theorem (p. 164).
6. Finding absolute extrema using critical numbers and endpoints.
7. Stating the Mean Value Theorem (p. 174)
8. Applying Rolle’s Theorem and the Mean Value Theorem.
9. Determining the intervals on which a function is increasing or decreasing using the first derivative.
10. Identifying relative maxima and minima using the first derivative test.

Below is a sampling of problems representative of the types you will see on the exam.

1. Let \( f(x) = x^3 - 3x + 1 \).
   (a) Find the derivative, \( f'(x) \).
   (b) For what values of \( x \) is \( f'(x) \) positive and for what values is it negative?
   (c) What does this tell you about the graph of \( f(x) \)?
2. Find \( f'(x) \) in each of the following cases.
   (a) \( f(x) = -2x \)
   (b) \( f(x) = x^2 - 3x + 1 \)
   (c) \( f(x) = \frac{2}{\sqrt{x}} \)
3. Find the slope of the line tangent to the graph of the lines above when \( x = 1 \).
4. Find and simplify the derivative of each function without using your calculator.
   (a) \( f(x) = \frac{x^2 + 1}{x+1} \)
   (b) \( g(x) = x \sin(x^2) \)
   (c) \( h(x) = \sqrt{x^2 - 3} \)
5. Find the second derivative of each of the functions above without using your calculator.
6. Are there any points on the graph of \( y = \sqrt{x^2 + 1} \) where the tangent line is horizontal? If so, find all such points.
7. Find all points \((x, y)\) at which the tangent line to the graph of \( y = x^3 + x + 1 \) has a slope of 13.
8. Use implicit differentiation to find the slope of the line tangent to each graph at the point indicated.
   (a) \( x^2y = 4 \) at \((2, 1)\)
   (b) \( 2x^3y - 3y^2 = -8 \) at \((1, 2)\)
   (c) \( x^2 + y^2 = 5 \) at \((8, 1)\)

9. A projectile is shot upward from the surface of the earth. The position of the object (above the earth) after \( t \) seconds is given by the equation \( s(t) = -4.9t^2 + 120t \).
   (a) Find the average speed of the object during the first 5 seconds of its flight.
   (b) Is there a point during the first 5 seconds when the projectiles’ instantaneous velocity is exactly equal to this average? Why or why not?
   (c) Find the instantaneous velocity of the object when \( t = 2 \) and when \( t = 5 \).
   (d) When the projectile reaches its highest point, it will momentarily have a velocity of zero. When does this happen, and what is that highest point reached?

10. At a sand and gravel plant, sand is falling off a conveyor belt and onto a conical pile at a rate of 10 cubic feet per minute. The diameter of the base of the cone is approximately three times the altitude. At what rate is the height of the pile changing when the pile is 15 feet tall? Recall that the volume formula for a cone is \( V = \frac{\pi r^2 h}{3} \).

11. A balloon rises at a rate of 3 meters per second from a point on the ground 30 meters from an observer. Find the rate of change of the angle of elevation of the balloon from the observer when the balloon is 30 meters above the ground.

12. Locate the absolute extrema of the functions below on the closed interval given.
   (a) \( g(x) = \sqrt{x} \) on \([-1, 1]\)
   (b) \( h(s) = -s^2 + 3s \) on \([0, 3]\)
   (c) \( f(x) = x^2 - 2x \) on \([-1, 2]\), on \((1, 3]\), and on \([1, 4]\).

13. Determine of Rolle’s Theorem can be applied on the interval given.
   (a) \( f(x) = x^{\frac{2}{3}} - 1 \) on \([-8, 8]\).
   (b) \( f(x) = x^{\frac{3}{4}} \) on \([-1, 1]\).
   (c) \( f(x) = \tan x \) on \([0, \pi]\).

14. Determine if the Mean Value Theorem can be applied to \( f \) on the closed interval \([a, b]\). If it can, find all values of \( c \) in the open interval \((a, b)\) such that \( f'(c) = \frac{f(b) - f(a)}{b - a} \).
   (a) \( f(x) = \frac{x + 1}{x} \) on \([\frac{1}{2}, 2]\).
   (b) \( f(x) = \cos x \) on \([\frac{\pi}{2}, \frac{3\pi}{2}]\).
   (c) \( f(x) = x^3 \) on \([0, 1]\).

15. Two bicyclists begin a race at 8:00 a.m. Both finish the race 2 hours and 15 minutes later. Prove that at some time during the race, the bicyclists are traveling at the same velocity.

16. Identify intervals on which the following functions are increasing or decreasing. Use the first derivative test to locate relative maxima and minima.
   (a) \( f(x) = 4x^3 - 48x + 13 \)
   (b) \( f(x) = 3x^5 - 5x^3 \)
   (c) \( f(x) = (1 + 3x)^{\frac{2}{3}} \)
   (d) \( f(x) = 2 \tan 2x^2 + 1 \)