Exam III Review Sheet
MATH 250, Spring 2003

This exam will cover sections 7.1-7.5, 8.1-8.4, and 8.6-7 in your text. You should know general terms and definitions from each of these sections, review the homework given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Definition and properties of relations
2. n-ary relations and operations (select, project, etc.)
3. Representing relations using matrices or digraphs
4. Closures of relations, including transitive closures
5. Equivalence relations, equivalence classes, and partitions
6. Graph classification, terminology, and examples
7. Representing Graphs using adjacency or incidence matrices
8. Determining if graphs are isomorphic
9. Definitions and applications of paths and connectivity
10. Euler and Hamiltonian paths
11. Planar graphs and graph colorings

Below is a list of sample problems. This list is not all-inclusive, but does represent the types of problems you will see on the exam.

1. Determine which of the properties: Reflexive, Symmetric, Antisymmetric, and Transitive, is enjoyed by each of the following relations.
   (a) \( R_1 = \{(a, b) \leftrightarrow a \text{ is a student of teacher } b\} \)
   (b) \( R_2 = \{(\spadesuit, \spadesuit), (\clubsuit, \spadesuit), (\heartsuit, \spadesuit), (\spadesuit, \heartsuit), (\heartsuit, \heartsuit)\} \)
   (c) \( R_3 = \{(n, m) \mid n = m^2\} \)
   (d) \( R_4 = \{(n, m) \mid n \equiv m (\text{mod } 3)\} \)

2. Using the relations above, find each of the following new relations.
   (a) \( R_3 \cup R_2 \)
   (b) \( R_3 \circ R_2 \)
   (c) \( R_1^{-1} \)
   (d) \( R_2 \) if \( R_2 \) is a relation on the set \{\spadesuit, \clubsuit, \heartsuit, \diamondsuit\}

3. Suppose that the following relations are given. Give the sequence of relational operations on the relation required to produce each new relation described below.

<table>
<thead>
<tr>
<th>student_id</th>
<th>name</th>
<th>major</th>
<th>class</th>
<th>club_name</th>
<th>student_id</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>david</td>
<td>math</td>
<td>freshman</td>
<td>math club</td>
<td>10</td>
</tr>
<tr>
<td>15</td>
<td>sarah</td>
<td>comp. sci.</td>
<td>senior</td>
<td>math club</td>
<td>15</td>
</tr>
<tr>
<td>14</td>
<td>david</td>
<td>engineering</td>
<td>junior</td>
<td>cs club</td>
<td>15</td>
</tr>
</tbody>
</table>

   (a) The relation containing names of all students
   (b) The relation containing the names of all math majors
   (c) The relation containing the names of all senior cs club members
   (d) The relation containing the names of all members of both the cs and math clubs
4. Represent each of the following relations as a matrix and a digraph. Use the matrix representation to determine if the relation is reflexive and/or symmetric. Use the digraph representation to check transitivity. Assume that the relations are all on the set \{a, b, c, d, e\}

(a) \{(a, a), (a, c), (b, a), (b, c), (b, d), (c, a), (c, b), (d, a), (e, c)\}
(b) \{(a, b), (d, c), (e, a), (b, a), (c, a), (a, c), (e, d), (a, e), (a, c)\}
(c) \{(a, b), (d, c), (b, d), (a, d), (b, c), (a, c)\}

5. Use the methods seen in your text to find the reflexive, symmetric, and transitive closures of each of the relations given above.

6. Consider the relation \(R = \{(x, y) \mid x - y \text{ is an integer}\}\).
   (a) Show that \(R\) is an equivalence relation.
   (b) Describe the equivalence classes \([1]_R\) and \([1/2]_R\).

7. List three different partitions of the set \{-3, -2, -1, 0, 1, 2, 3\} and give the equivalence relation which goes with each partition.

8. Draw each of the following graphs: \(K_6\), \(K_{2,5}\), \(C_7\), \(Q_3\), and \(W_6\).

9. For each of the graphs above, determine:
   (a) if the graph is simple, pseudo, a multi-graph, and a directed or undirected graph.
   (b) the adjacency and incidence (in the last two cases) matrices of the graph.
   (c) if the graphs have Hamilton or Euler paths or circuits.
   (d) if the graphs are strongly or weakly connected, identifying connected components and cut edges/vertices.
   (e) if the graphs are planar, giving a planar representation when appropriate.
   (f) the chromatic number of the graph.

10. Find the in and out degree of each vertex in the first graph given.

11. Determine if the last two graphs are isomorphic. If they are, give an isomorphism. If they are not, give a justification.

12. Use the adjacency matrix to determine the number of paths of length 3 between any pair of vertices in the last graph given above.

13. Use Kuratowski’s Theorem to determine if graphs 23-25 on page 612 of your text are planar.

14. Give an example of a graph with neither a Hamiltonian circuit or Euler circuit.