Exam I Review Sheet  
MATH 250, Spring 2007

This exam will cover sections 1.1-1.6 and 2.1-2.4 in your text. You should know general terms and definitions from each of these sections, review the homework given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Translation between English and propositional logic.
2. The construction of truth tables for compound propositional statements.
3. Showing logical equivalences with truth tables or strings of equivalences and identities from table 6 (page 24, know the names) and \( p \rightarrow q \equiv \neg p \lor q \).
4. Translating between English and predicate logic and working with quantifiers.
5. Performing symbolic proofs using rules of inference from table 1 (page 66) and table 2 (page 70). You do not need to know the names of these rules.
6. Proving basic topics in algebra via direct, indirect, contradiction, equivalence, or case-by-case methods.
7. Working with set definitions and properties, power sets, and the Cartesian product of sets.
8. Showing set operations are the same using the identities in table 1 (page 124, know the names), logical equivalences, Venn diagrams, or set inclusion tables.
9. Working with generalized unions and/or intersections.
10. Utilizing the definition of a function, injective and surjective functions, and function composition to analyze existing functions or create and work with new functions.
11. Working with sequences and sums, including the summation formulae from table 2 on page 157.
12. Comparing the cardinality of sets, both finite and infinite.

Below is a list of sample problems. While not all-inclusive, the list represents typical exam problems.

1. Let \( p \) : You get an A on the final, \( q \) : You do every exercise in this book, and \( r \) : You get an A in the class. Translate the following between English and propositional logic and construct truth tables.
   (a) You get an A in the class, but you do not do every exercise in the book.
   (b) You get an A on the final, and do every exercise in the book, but you don’t get an A in the class.
   (c) You get an A in the class if and only if you do every exercise and get an A on the final.
   (d) \( \neg(p \lor q) \)
   (e) \( (p \land \neg q) \rightarrow r \)
   (f) \( \neg(p \land r) \lor (p \land q) \)

2. Determine if the following compound propositions are logically equivalent using known equivalences.
   (a) \( (p \rightarrow q) \land (p \rightarrow r) \) and \( p \rightarrow (q \land r) \)
   (b) \( \neg p \rightarrow (q \rightarrow r) \) and \( q \rightarrow (p \lor r) \)
   (c) \( (\neg q \land (p \rightarrow q)) \rightarrow \neg p \) and \( T \) (i.e. the first expression is a tautology).

3. Determine if the following pairs of quantified predicate statements are logically equivalent, where \( A \) is a proposition not involving quantifiers.
   (a) \( (\forall x P(x)) \lor A \) and \( \forall x (P(x) \lor A) \)
   (b) \( \exists x (P(x) \land Q(x)) \) and \( \exists x P(x) \land \exists x Q(x) \)
   (c) \( \exists x (P(x) \lor Q(x)) \) and \( \exists x P(x) \lor \exists x Q(x) \)
   (d) \( \neg \forall x \exists y \exists z (P(x, y) \lor Q(x, z)) \) and \( \exists x \forall y \forall z (\neg P(x, y) \land \neg Q(x, z)) \)
4. Use quantifiers and predicates with more than one variable to express these statements.
   (a) There is a student in this class who speaks German.
   (b) Every student in this class plays some musical instrument.
   (c) Some student in this class plays the oboe and speaks German.
   (d) Some student in this class grew up in the same town as exactly one other student in this class
       and went to high school with every other student in this class.

5. Perform the following proofs symbolically, if they are valid. If they are not valid, state the reason.
   (a) “Randy works hard,” “If Randy works hard, then he is a dull boy,” and “If Randy is a dull boy,
       then he will not get the job” imply “Randy will not get the job.”
   (b) “Every C.S. major takes Discrete Math,” “All C.S. majors are in the C.S. club,” and “Natasha
       is not in the C.S. club,” therefore “Natasha does not take Discrete Math”

6. Prove that if \( m, n \in \mathbb{Z} \) and \( mn = 1 \) then either \( m = n = 1 \) or \( m = n = -1 \) using the following methods.
   (a) Direct Proof
   (b) Indirect Proof
   (c) Contradiction

7. Answer each of the following questions related to the sets \( A = \{x, y, z\} \) and \( B = \{1, 2\} \).
   (a) What is the set \( \mathcal{P}(A) \)?
   (b) What is the cardinality of \( \mathcal{P}(\mathcal{P}(A)) \)?
   (c) What are the elements of \( A \times B \)?
   (d) What are the sets \( (A\backslash B \cup B\backslash A) \), \( (A\backslash B \cap B\backslash A) \), and \( A \oplus B \)?

8. Suppose that \( f \) and \( g \) are bijections form \( \mathbb{N} \) to \( \mathbb{N} \). Show that:
   (a) \( f \circ g \) is an injection
   (b) \( f \circ g \) is a bijection
   (c) \( f[S \cap T] = f[S] \cap f[T] \) for \( S, T \subseteq \mathbb{N} \).

9. Provide a simple rule for each sequence, and find the value of each sum.
   (a) \( 1, 0, 2, 0, 4, 0, 8, 0, \ldots \)
   (b) \( 2, 3, 7, 25, 121, 721, 5041, \ldots \)
   (c) \( \sum_{j=0}^{8} (1 + (-1)^j) \)
   (d) \( \sum_{i=1}^{3} \sum_{j=0}^{2} i^2j^3 \)

10. Determine whether each of these sets is countable or uncountable. For those that are countable, exhibit
    a one-to-one correspondence between the set of natural numbers and that set.
    (a) the negative integers
    (b) the even integers
    (c) the real numbers between 0 and \( \frac{1}{2} \)
    (d) integers that are multiples of 7