Exam II Review Sheet
MATH 281, Spring 2004

This exam will cover sections 5.6-6.3 in your text. You should know general terms and definitions from each of these sections, review the homework given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Setting up and solving growth and decay problems.
2. Solving differential equations by separating variables.
3. Identifying and solving homogeneous differential equations.
4. Evaluating, graphing, and defining inverse trigonometric functions.
5. Differentiating inverse trigonometric functions.
6. Finding integrals involving inverse trigonometric functions.
7. Evaluating, graphing, and proving identities for hyperbolic functions.
8. Differentiating and integrating hyperbolic functions.
10. Finding the area between two or more curves.
11. Finding the volume of a solid of rotation using the disk method.
12. Finding the volume of a solid of rotation using the shell method.

Below is a sampling of problems representative of the types you might see on the exam.

1. The number of bacteria in a culture increases from 600 to 1800 in two hours. Assuming that the rate of increase is proportional to the number of bacterial present, find a formula for the number of bacteria after \( t \) hours. What is the number of bacteria after 4 hours?

2. According to Newton’s law of cooling, the rate at which a body cools is proportional to the difference between the temperature of the body and the temperature of the surrounding medium. A roast toferkey is taken from an oven when its temperature has reached 180°F and is placed on a table in a room where the temperature is 75°F.
   (a) If the temperature of the toferkey is 150°F after half an hour, what will the temperature be after 45 minutes?
   (b) When will the toferkey have cooled to 100°F?

3. Warfarin is a drug used as an anticoagulant. After stopping administration of the drug, the quantity remaining in a patient’s body decreases at a rate proportional to the quantity remaining. The half-life of the warfarin concentration in the blood is 37 hours.
   (a) Sketch a graph of the quantity, \( Q \), of warfarin in a patient’s body as a function of time, \( t \), since stopping administration of the drug. Mark the 37-hour point on your graph.
   (b) Formulate a differential equation satisfied by \( Q \).
   (c) How many days does it take the drug level in the body to be reduced to 10% of the original level?

4. Solve each differential equation.
   (a) \( y' + 3y = 3 \)
   (b) \( y' + 2xy = x \)
   (c) \( y' = x(1 + y^2) \)
   (d) \( y' = \frac{x^3 + y^3}{xy^2} \)
5. Is it true that \( \sin^{-1}(\sin 2) = 2 \)? Why or why not? What about \( \cos^{-1}(\cos 2) = 2 \)?

6. Find the derivative \( \frac{dy}{dx} \) for each of the following.

   (a) \( y = \frac{\sin^{-1} x}{x + \tan^{-1} x} \)

   (b) \( y = \cosh x^2 \)

   (c) \( y = \tan^{-1}(x^2 + 1) \)

   (d) \( y = \sin^{-1}(\cos 2x) \)

   (e) \( y = \sinh x \cosh x \)

   (f) \( y = \frac{1 + \cosh x}{1 - \cosh x} \)

7. Find each of the following integrals

   (a) \( \int_0^1 \frac{1 + x}{2x^2 + 3} \, dx \)

   (b) \( \int \frac{2}{x^2 + 2x + 3} \, dx \)

   (c) \( \int \frac{1}{x\sqrt{4x^2 - 16}} \, dx \)

   (d) \( \int \frac{x}{\sqrt{16 - 4x^2}} \, dx \)

   (e) \( \int \frac{\sinh \sqrt{x}}{\sqrt{x}} \, dx \)

   (f) \( \int \frac{\cosh \ln x}{x} \, dx \)

   (g) \( \int \frac{e^{\sinh x}}{\text{sech} x} \, dx \)

8. Use the formula \( \sinh x = \frac{e^x - e^{-x}}{2} \) to derive a formula for \( \sinh^{-1} x \). Give the domain of the inverse.

9. Find the area bounded by the graph of \( y = \frac{x^2}{1 + x^6} \), the \( x \)-axis, and the lines \( x = 0 \) and \( x = 1 \).

10. Find the volume of the solid formed by rotating the region bounded by \( y = \frac{1}{\sqrt{1 + x^2}} \), \( x = 0 \) and \( x = 1 \) about the \( x \)-axis.

11. Find the volume of the solid formed by rotating the region between \( y = x^2 \) and \( y = 4 \) about

    (a) the \( y \)-axis.

    (b) the line \( y = 4 \).

    (c) the line \( x = 2 \).

12. A hole of diameter \( d \) is drilled through a spherical solid of radius \( r \) so that the axis of the hole is a diameter of the sphere. Find the volume of the solid that remains.

13. A solid has as its base the region in the \( xy \)-plane bounded by the graphs of \( y = x \) and \( y = x^2 \). If every cross section by a plane perpendicular to the \( y \)-axis is a half disk, find the volume of the solid.