Exam II Review Sheet
MATH 281, Winter 2006

This exam will cover sections 5.6-7.3 in your text. You should know general terms and definitions from each of these sections, review the homework given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Evaluating, graphing, and defining inverse trigonometric functions.
2. Differentiating inverse trigonometric functions.
3. Finding integrals involving inverse trigonometric functions.
4. Evaluating, graphing, and proving identities for hyperbolic functions.
5. Differentiating and integrating hyperbolic functions.
6. Working with inverse hyperbolic functions.
7. Setting up and solving growth and decay problems.
8. Solving differential equations by separating variables.
10. Finding the area between two or more curves.
11. Finding the volume of a solid of rotation using the disk method.
12. Finding the volume of a solid of rotation using the shell method.

Formulas you should know (from these sections):

\[ \sinh x = \frac{e^x - e^{-x}}{2} \quad \cos h x = \frac{e^x + e^{-x}}{2} \]
\[ \frac{d}{dx} \sinh u = (\cosh u)u' \quad \frac{d}{dx} \cosh u = (\sinh u)u' \]
\[ \int \sinh u \, du = \cosh u + C \quad \int \cosh u \, du = \sinh u + C \]
\[ \frac{d}{dx} \arcsin u = \frac{u'}{\sqrt{1-u^2}} \quad \int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + C \]
\[ \frac{d}{dx} \arctan u = \frac{u'}{1+u^2} \quad \int \frac{du}{a^2+u^2} = \frac{1}{a} \arctan \frac{u}{a} + C \]
\[ \frac{d}{dx} \tanh^{-1} u = \frac{u'}{1-u^2} \]
\[ A = \int_a^b [f(x) - g(x)] \, dx \quad V = \pi \int_a^b [R(x)]^2 \, dx \] (disk)
\[ V = 2\pi \int_a^b p(x)h(x) \, dx \] (shell)

Below is a sampling of problems representative of the types you might see on the exam.

1. The number of bacteria in a culture increases from 600 to 1800 in two hours. Assuming that the rate of increase is proportional to the number of bacterial present, find a formula for the number of bacteria after \( t \) hours. What is the number of bacteria after 4 hours?

2. According to Newton's law of cooling, the rate at which a body cools is proportional to the difference between the temperature of the body and the temperature of the surrounding medium. A roast toferkey is taken from an oven when its temperature has reached 180°F and is placed on a table in a room where the temperature is 75°F.

   (a) If the temperature of the toferkey is 150°F after half an hour, what will the temperature be after 45 minutes?

   (b) When will the toferkey have cooled to 100°F?
3. Solve each differential equation.
   (a) $y' + 3y = 3$
   (b) $y' + 2xy = x$
   (c) $y' = x(1 + y^2)$
   (d) $y' = \frac{x^4 + y^2}{xy^2}$

4. Is it true that $\sin^{-1}(\sin 2) = 2$? Why or why not? What about $\cos^{-1}(\cos 2) = 2$?

5. Find the derivative $\frac{dy}{dx}$ for each of the following.
   (a) $y = \frac{\sin^{-1} x}{x + \tan^{-1} x}$
   (b) $y = \cosh x^2$
   (c) $y = \tan^{-1}(x^2 + 1)$
   (d) $y = \sin^{-1}(\cos 2x)$
   (e) $y = \sinh x \cosh x$
   (f) $y = \frac{1 + \cosh x}{1 - \cosh x}$

6. Find each of the following integrals
   (a) $\int_0^1 \frac{1 + x}{2x^2 + 3} \, dx$
   (b) $\int \frac{2}{x^2 + 2x + 3} \, dx$
   (c) $\int \frac{e^{\sinh x}}{\text{sech} \, x} \, dx$
   (d) $\int \frac{x}{\sqrt{16 - 4x^2}} \, dx$
   (e) $\int \frac{\sinh \sqrt{x}}{\sqrt{x}} \, dx$
   (f) $\int \frac{\cosh \ln x}{x} \, dx$

7. Use the formula $\sinh x = \frac{e^x - e^{-x}}{2}$ to derive a formula for $\sinh^{-1} x$. Give the domain of the inverse.

8. Find the area bounded by the graph of $y = \frac{x^2}{1 + x^2}$, the x-axis, and the lines $x = 0$ and $x = 1$.

9. Find the volume of the solid formed by rotating the region bounded by $y = \frac{1}{\sqrt{1 + x^2}}$, $x = 0$ and $x = 1$ about the x-axis.

10. Find the volume of the solid formed by rotating the region between $y = x^2$ and $y = 4$ about
    (a) the y-axis.
    (b) the line $y = 4$.
    (c) the line $x = 2$.

11. A hole of diameter $d$ is drilled through a spherical solid of radius $r$ so that the axis of the hole is a diameter of the sphere. Find the volume of the solid that remains.

12. A solid has as its base the region in the xy-plane bounded by the graphs of $y = x$ and $y = x^2$. If every cross section by a plane perpendicular to the y-axis is a half disk, find the volume of the solid.