This exam will cover sections 7.4-8.5 in your text. You should know general terms and definitions from each of these sections, review the homework given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Finding arc lengths of two dimensional curves.
2. Finding the area of a surface of revolution.
3. Computing the work done by a constant or variable force.
4. Computing moments and the center of mass for a one-dimensional system.
5. Computing moments and the center of mass (centroid) for a planar lamina.
6. Finding the volume of a solid of rotation using the Theorem of Pappus.
7. Computing the fluid force on a vertical surface.
8. Utilization of basic integration rules and “tricks”.
9. Stating and proving the integration by parts theorem.
10. Performing integration by parts.
11. Special formulas for trigonometric integration.
12. Integration by trigonometric substitution.
13. Partial fraction expansion.

Below is a sampling of problems representative of the types you might see on the exam.

1. Find the length of the arc of the circle with equation \( x^2 + y^2 = 4 \) between the points \((1, \sqrt{3})\) and \((1.5, \sqrt{1.75})\). You may use your calculator to get an approximate value of the appropriate integral.

2. Find the length of the segment of the curve with equation \( y = \frac{2}{3}x^2 \) for \( x \in [3, 15] \).

3. The arc of the parabola \( y^2 = x \) joining the points \( (1, 1) \) and \( (4, 2) \) is revolved about the \( x \)-axis. Find the area of the resulting surface.

4. Find the work done when a 100 lb woman climbs a 6 foot ladder in: 5 seconds, 10 seconds, and 20 seconds.

5. A freight elevator weighing 3000 lb. is supported by a 12 ft. cable weighing 15 lbs. per linear foot. find the work required to lift the elevator 10 ft. by winding the cable onto a winch.

6. A water trough is 10 feet long. The ends of this trough are equilateral triangles with sides measuring 2 feet. If this trough is full of water weighing 62.4 lb/ft\(^3\), set up the integral that gives the work required to pump all of the water in the trough over the top. You need not evaluate this integral.

7. The region above the \( x \)-axis and below the graph of \( y = \sqrt{1-x^2} \) is revolved about the line \( y = -1 \). Use the theorem of Papus to find the volume of the resulting solid.

8. A cylindrical oil tank 6 feet in diameter and 10 feet long is lying on its side. If the tank is half full of oil weighing 58 pounds per cubic foot, find the force exerted by the oil on one end of the tank.

9. Find the centroid of the region above the \( x \)-axis and inside the unit circle.

10. Use the formula for arc length to find the length of the curve \( y = x^2 \) between \( x = 0 \) and \( x = 1 \).

11. Derive the integration formula \( \int \cot x \, dx = \ln |\sin x| + C \).
12. State and prove the integration by parts theorem. (You may assume the product rule holds.)

13. Find each integral using appropriate integration techniques. Do not use your calculator’s integration abilities.

(a) \[ \int \frac{\cot \sqrt{x}}{\sqrt{x}} \, dx \]
(b) \[ \int_{-1}^{1} \frac{e^t}{e^t + 1} \, dt \]
(c) \[ \int \frac{2}{x^2 + 2x + 3} \, dx \]
(d) \[ \int \frac{\cos x}{\sin x + 5} \, dx \]
(e) \[ \int \frac{1}{\sqrt{16 - 4x^2}} \, dx \]
(f) \[ \int \frac{1}{x\sqrt{4x^2 - 16}} \, dx \]
(g) \[ \int \frac{x}{\sqrt{16 - 4x^2}} \, dx \]
(h) \[ \int_0^x \frac{\sin 2x}{1 + \sin^2 x} \, dx \]
(i) \[ \int x \sin x \, dx \]
(j) \[ \int e^x \sin x \, dx \]
(k) \[ \int_0^1 x(x^2 + 2)^{10} \, dx \]
(l) \[ \int_0^1 x(x + 2)^{10} \, dx \]
(m) \[ \int e^{2x} \sin e^x \, dx \]
(n) \[ \int \frac{2}{x^2 - 1} \, dx \]
(o) \[ \int \frac{x^3}{x^2 - 1} \, dx \]
(p) \[ \int \frac{x + 1}{(x^2 + 1)(x - 1)} \, dx \]
(q) \[ \int \frac{5x^3 - 3x^2 + 7x - 3}{(x^2 + 1)^2} \, dx \]
(r) \[ \int \frac{1}{x^2 - a^2} \, dx \]
(s) \[ \int x^3 \sqrt{1 - x^2} \, dx \]