Exam II Review Sheet  
MATH 282, Autumn 2008

This exam will cover sections 9.9-10.6 and Appendix E in your text. You should know general terms and definitions from each of these sections, review the homework given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Be able to find geometric power series, Taylor series, and Maclaurin series for a given function.
2. Find the interval on which a Taylor series for a function $f$ converges to $f$.
3. Derive Taylor series from a list of known functions: $e^x$, $\frac{1}{x}$, and $\sin x$.
4. Sketch the graph of and find characteristics of a parabola, ellipse, or hyperbola based on an equation.
5. Write the equation for a conic based on given characteristics or a graph.
6. Translate between parametric and rectangular equations.
7. Sketch the graph of a curve represented by parametric equations and determine its orientation.
8. Differentiate parametric equations to find the slope and concavity of the represented curve.
9. Determine the arc length of a curve represented by parametric equations.
10. Find the area of a surface of revolution for a curve represented by parametric equations.
11. Convert points and equations between rectangular and polar form.
12. Sketch the graph (without calculator) of a simple polar equation.
13. Determine the slope of tangent lines to a polar curve at a given point.
14. Find the area of a region described using polar equation(s).
15. Find the arc length or surface of revolution of a curve represented by polar equations.
16. Identify conic sections in polar form, and write polar equations for given conic sections.
17. Rotate axes to identify conic sections from a general second degree equation and sketch its graph.
18. Use the discriminant to quickly identify the conic section represented by a general second degree equation in the $xy$-plane.

Below is a sampling of problems representative of the types you might see on the exam.

1. Let $g(x) = \frac{2}{3^{x^2}}$. Remember that a power series “comes with” a radius of convergence.
   (a) Find the geometric power series centered at 0 for this function.
   (b) Find the power series centered at 0 for $g'(x)$ and $\int g(x) \, dx$.

2. Determine the first four terms of the Maclaurin series for $e^{2x}$ by:
   (a) using the definition and the formula $a_n = \frac{f^{(n)}(0)}{n!}$.
   (b) replacing $x$ by $2x$ in the series for $e^x$.
   (c) multiplying the series for $e^x$ by itself.

3. Find an equation of:
   (a) parabola with vertex $(-3, 4)$ and focus $(1, 4)$
   (b) hyperbola with foci $(0, \pm 3)$ and one vertex at $(0, -2)$
   (c) ellipse with vertices at $(0, 8)$ and $(0, 2)$ and $c = \sqrt{5}$
   (d) set of points the sum of whose distances from $(4, -3)$ and $(-4, -3)$ is 12.
   (e) set of all points equidistant from the line $x = -3$ and the point $(-1, 0)$.
   (f) parabola with focus $(1, 4)$ and vertex $(-3, 4)$. 

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4. Find the area of one petal of the rose \( r = 5 \sin 4 \theta \).

5. Find the distance between the polar points \((2, \pi/4)\) and \((3, \pi)\).

6. Give parametric equations for the graph of the ellipse with \(a = 3\) and \(b = 4\) centered at the origin.

7. Sketch and identify each graph. Give the foci, vertices, and directrix (if appropriate), and graph the asymptotes (if appropriate).

   (a) \[ 9x^2 + 4y^2 = 36 \]
   (b) \[ x^2 - 4y^2 = 36 \]
   (c) \[ x^2 + 9y^2 - 18y + 8 = 0 \]
   (d) \[ x^2 - 9y^2 + 18y + 27 = 0 \]
   (e) \[ 4x^2 - y^2 = 16 \]
   (f) \[ r = \frac{1}{2+2 \cos \theta} \]
   (g) \[ r = \frac{1}{2+\sin \theta} \]
   (h) \[ r = \frac{4}{2+\sin \theta} \]

8. Give parametric equations for each curve.

   (a) The line passing through the points \((1, 2)\) and \((4, 3)\).
   (b) The circle with radius 2 and center \((3, 1)\).

9. Let \(C\) be given by the parametric equations \(x = 5t^2, y = 2t^3\) for \(t \in [0, 1]\). Give the exact length of \(C\).

10. Convert \((3, \pi/2)\) to rectangular coordinates, and \((-5, 2)\) to polar coordinates.

11. Sketch the graph of \(r = 1 - 2 \cos \theta\). Find the slope of the tangent line to this graph at \(\theta = \pi/2\).

12. Let \(C_1\) be the curve defined by \(x = 2 \sin t\) and \(y = 3 \cos t\) for \(0 \leq t \leq \pi\) and \(C_2\) the curve defined by \(x = \cos^3 t, y = \sin^3 t\) for \(0 \leq t \leq \pi/2\).

   (a) Sketch the graphs of \(C_1\) and \(C_2\).
   (b) Find the slope of the tangent line at the points \((1, \frac{3\sqrt{3}}{2})\) for \(C_1\) and \((\frac{1}{8}, \frac{3\sqrt{2}}{8})\) for \(C_2\).
   (c) What happens to the slope of the tangent line to \(C_2\) as \(t\) approaches 0 and \(\pi/2\)?
   (d) Find the length of \(C_1\) and \(C_2\).
   (e) Find an equation for \(C_1\) in terms of \(x\) and \(y\).

13. Carefully sketch each graph.

   (a) \(r = 2 \sin 2\theta\)
   (b) \(r = 2 + \cos \theta\)
   (c) \(r = 1 + 2 \sin \theta\)
   (d) \(r = 3 \sin 30^\circ\)
   (e) \(\theta = -\frac{\pi}{3}\)

14. Find the slope of the line tangent to the graph of \(r = 4 \cos \theta\) at the point \((r, \theta) = (2\sqrt{2}, \pi/4)\).

15. Find the area contained inside the graph of \(r = 3 \cos \theta\) and outside the graph of \(r = 1 + \cos \theta\).

16. Sketch the graphs of the curves given by the following parametric equations.

   (a) \(x = t, y = \sin t\) for \(-\pi \leq t \leq \pi\)
   (b) \(x = t^2, y = \sin t^2\) for \(-\pi \leq t \leq \pi\)

17. Rotate the axes to eliminate the \(xy\)-term in the equation \(xy - 4 = 0\). Give the new equation and then identify and sketch the graph of this conic section.

18. Find the angle \(\theta\) through which the axes need to be rotated in order to eliminate the \(xy\)-term in \(7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0\), and find the new equation.

19. Use the discriminant to determine the type of conic represented by each of the equations below.

   (a) \(2x^2 - xy + 4y^2 + x - y + 7 = 0\)
   (b) \(3x^2 + 6xy + 3y^2 + 7x - 2y - 10 = 0\)
   (c) \(x^2 - 4xy + 2y^2 + 6y - 1 = 0\)