This exam will cover sections 9.1-12.5 in your text, as well as Appendix E. You should know general terms and definitions from each of these sections, review the homework and previous exams, and pay particular attention to the subjects and practice problems mentioned below.

1. Chapter 9
   (a) Know the definition of the limit of a sequence as found in section 9.1.
   (b) Properties of sequences such as: boundedness, monotonicity, etc.
   (c) Be able to identify and find the sum of a geometric or telescoping series.
   (d) Correctly applying the following series tests to determine convergence/divergence:
      i. $n$th term test
      ii. integral test
      iii. $p$-series test
      iv. direct comparison test
      v. limit comparison test
      vi. alternating series test
      vii. ratio test
      viii. root test
   (e) Determining absolute or conditional convergence
   (f) Determining the interval of convergence of a power series or its derivative or integral.
   (g) Finding power series (Geometric, Taylor, Maclaurin) for a given function.

2. Chapter 10 & Appendix E
   (a) Translate between the equation and graph of an ellipse, parabola, and hyperbola.
   (b) Translate between parametric and rectangular equations.
   (c) Find the graph, slope, concavity, and arc length of a curve defined by parametric equations.
   (d) Convert points and equations between rectangular and polar coordinate systems.
   (e) Determine slope, area, and arc length for a curve or region defined by polar equations.
   (f) Identify and find the polar equations for conic sections.
   (g) Rotate axes to eliminate $xy$-terms and sketch the graph of a general second degree equation.
   (h) Identifying a conic based on the discriminant of the general second degree equation.

3. Chapter 11
   (a) Vector arithmetic including dot- and cross-products.
   (b) Be able to prove theorem 11.2 ($\|cv\| = |c|\|v\|$).
   (c) Use the dot-product to: find the angle between vectors, show vectors are orthogonal, or find a projection of one vector onto another.
   (d) Use the cross-product to: find an orthogonal vector, find the area of a parallelogram, or find the volume of a parallelepiped.
   (e) Find equations for lines and planes in space, and sketch those lines or planes.
   (f) Find angles and distances between lines, planes, and points in space as done in your text.
   (g) Identify and sketch cylindrical and quadratic surfaces in space.
   (h) Convert points and equations between rectangular, cylindrical, and spherical coordinate systems.

4. Chapter 12
   (a) Evaluate, differentiate, and integrate vector-valued functions.
   (b) Relate position, velocity, and acceleration vectors.
(c) Use vectors to solve projectile-motion problems.
(d) Find unit tangent and principal unit normal vectors to a smooth curve represented by a vector-valued function.
(e) Determine the tangential and normal components of an acceleration vector.
(f) Determine the arc length and arch length parameterization of a curve.
(g) Find the curvature of a smooth curve.

Below is a sample of problems representative of the types you might see from sections 12.3-12.5. See previous review sheets and exams for problems from sections 9.1-12.2 and Appendix E.

1. If an object’s position in space at time \( t \) is given by \( \mathbf{r}(t) = \langle 2t, t^2, 1 \rangle \), then:
   (a) give the location of the object at time \( t = 2 \).
   (b) give the speed of the object at time \( t = 2 \).
   (c) find the acceleration of the object at time \( t = 2 \).
   (d) in what direction is the object traveling at time \( t = 2 \)?

2. Consider the curve \( C \) defined by \( \mathbf{F}(t) = \langle t^2, t, 1 \rangle \) for \( t \in [0, 3] \).
   (a) Find the principal unit tangent and normal vectors at \( t = 1 \).
   (b) Sketch the graph of \( C \) and illustrate these vectors on your graph.
   (c) Set up an integral which gives the length of \( C \).

3. Consider the curve \( C \) described by \( \mathbf{r}(t) = \mathbf{i} + t\mathbf{j} + (\ln t)\mathbf{k} \) for \( t > 0 \).
   (a) Give parametric equations for \( C \).
   (b) Sketch the graph of \( C \).
   (c) Find \( \mathbf{T}(1) \) and \( \mathbf{N}(1) \) and illustrate these on your graph.

4. Let \( C \) be described by \( \mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + t^2\mathbf{k} \) for \( t \in [-1, 1] \).
   (a) Set up, but do not evaluate, the integral giving the length of \( C \).
   (b) Find the curvature of \( C \) at \( t = 0 \).

5. Suppose that a point is moving along the path given by \( \mathbf{r}(t) = 4t\mathbf{i} + (3\cos t)\mathbf{j} + (3\sin t)\mathbf{k} \). Find the normal and tangential components of acceleration at time \( t = \frac{\pi}{2} \).

6. A point moves along a curve in such a way that its position at time \( t \) is \( \mathbf{r}(t) = (t-1)\mathbf{i} - (t-1)\mathbf{j} + e^{(t-1)}\mathbf{k} \) for \( t \in [0, 2] \).
   (a) Sketch the path of this point, and illustrate the velocity and acceleration vectors corresponding to \( t = 1 \).
   (b) Find the tangential and normal components of the acceleration vector at \( t = 1 \).

7. A projectile is fired from ground level at an angle of 20° with the horizontal. Find the minimum initial velocity if the projectile has a range of 80 meters.