Exam I Review Sheet
MATH 282, Spring 2006

This exam will cover sections 9.1-9.7 in your text. You should know general terms and definitions from each of these sections, review the homework given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Finding the limit of a sequence \( \{a_n\} \)
2. The definition of the limit of a sequence
3. Identifying bounded and monotonic sequences
4. Working with partial sums and infinite series
5. Identifying and working with telescoping series
6. Identifying and working with geometric series
7. Correctly applying the \( n \)th term test
8. Correctly applying the integral test
9. Identifying and working with \( p \)-series
10. Correctly applying the direct and limit comparison tests
11. Correctly applying the alternating series test
12. Finding the maximum error when approximating the sum of an alternating series
13. Determining if a series converges absolutely or conditionally
14. Correctly applying the root and ratio tests
15. Finding \( n \)th Taylor and Maclaurin polynomials
16. Applying Taylor’s Theorem to find the maximum error in an approximation of a function

Below is a sampling of problems representative of the types you might see on the exam.

1. Suppose that the \( n \)th term of a sequence \( a_1, a_2, \ldots, a_n, \ldots \) is given by \( a_n = \frac{2n^2-1}{3n^2} \).
   (a) Find the 7th term of the sequence.
   (b) Does the sequence have an upper bound? If so, find an upper bound.
   (c) Does the sequence have a limit? If so find it.

2. Mark the following True or False
   (a) ____ Every bounded sequence converges
   (b) ____ If \( \lim_{n \to \infty} a_n = 0 \) then \( \sum_{n=1}^{\infty} a_n \) converges.
   (c) ____ If \( \sum_{n=1}^{\infty} a_n \) is a convergent series with positive terms, then \( \sum_{n=1}^{\infty} a_n^2 \) is a convergent series.
   (d) ____ Any series that is absolutely convergent is also conditionally convergent.

3. Find the sum of the series \( \sum_{n=2}^{\infty} (-1)^n \left( \frac{3}{4} \right)^n \).
4. Classify each of the following series as divergent, conditionally convergent, or absolutely convergent. Name the test which gives you your conclusion.

(a) \( \sum_{n=1}^{\infty} \frac{n}{6n+1} \)

(b) \( \sum_{n=1}^{\infty} \frac{\ln n}{n^3} \)

(c) \( \sum_{n=1}^{\infty} ne^{-n^2} \)

(d) \( \sum_{n=1}^{\infty} (-1)^n \frac{1}{\ln n} \)

(e) \( \sum_{n=1}^{\infty} \frac{2}{n^2+1} \)

(f) \( \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}} \)

5. Suppose that the partial sum of the first 10 terms of the series \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \) is used as an estimate for the sum of the series. How large can the error be?

6. If \( S_n = 1 + .4 + (.4)^2 + \ldots + (.4)^{n-1} \) find \( \lim_{n \to \infty} S_n \)

7. Use the function \( f(x) = \sqrt{1+x} \) to solve the following problems.
   (a) Find the Maclaurin series for \( f(x) \).
   (b) Suppose that the first four terms of the above series are used to estimate \( f(4) \). Use the remainder formula to find a worst-case estimate of the error.

8. Suppose that the \( n \)th term of a sequence is given by \( a_n = \frac{n^2}{n^2+1} \) for \( n = 1, 2, 3, \ldots \).
   (a) Give the 3rd term.
   (b) Does the sequence have a limit? If so, find it.

9. Consider the series \( \sum_{n=0}^{\infty} 2^n \).
   (a) Give the first three terms of the sequence of partial sums for this series.
   (b) Does the series converge? Give a justification for your answer.

10. Find the sum of the series \( \sum_{n=1}^{\infty} \left( \frac{1}{2^n} - \frac{1}{3^n} \right) \).

11. Find the 4th and 6th Taylor Polynomial for \( f(x) = \ln x \) at \( x = 1 \).