Exam III Review Sheet
MATH 282, Winter 2007

This exam will cover sections 11.1-12.2 and appendix E. You should know general terms and definitions from each of these sections, review the homework given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Rotate axes to identify conic sections from a general second degree equation and sketch its graph.
2. Use the discriminant to quickly identify the conic section represented by a general second degree equation in the \(xy\)-plane.
3. Vector arithmetic using vectors in component and/or unit vector format.
4. Be able to prove theorem 11.2 (\(\|cv\| = |c|\|v\|\))
5. Be able to find a unit vector in the same direction as a given vector.
6. Use vectors to find resultant force.
7. Compute the dot product of two vectors, and prove properties of the dot product (Theorem 11.4).
8. Use the dot product to find angles between vectors or show they are orthogonal.
9. Find the projection of a vector onto another, as well as the orthogonal component vector.
10. Finding work done by a force acting at an angle to the direction of movement.
11. Compute the cross product of two vectors, and prove properties of the cross product (Theorem 11.7).
12. Using cross products to find: orthogonal vectors, area of a parallelogram or parallelopiped, and torque.
13. Finding equations for lines and planes in space and sketching their graphs.
14. Determining angles and distances between lines and planes in space.
15. Identifying and sketching surfaces in space from a given equation.
16. Converting points and equations between rectangular, cylindrical, and spherical coordinates.
17. Evaluating vector-valued functions and finding their limits.
18. Differentiating and integrating vector-valued functions.

Below is a sampling of problems representative of the types you might see on the exam.

1. Rotate the axes to eliminate the \(xy\)-term in the equation \(xy - 4 = 0\). Give the new equation and then identify and sketch the graph of this conic section.
2. Find the angle \(\theta\) through which the axes need to be rotated in order to eliminate the \(xy\)-term in \(7x^2 - 6\sqrt{3}xy + 13y^2 - 16 = 0\), and find the new equation.
3. Use the discriminant to determine the type of conic represented by each of the equations below.
   (a) \(2x^2 - xy + 4y^2 + x - y + 7 = 0\)
   (b) \(3x^2 + 6xy + 3y^2 + 7x - 2y - 10 = 0\)
   (c) \(x^2 - 4xy + 2y^2 + 6y - 1 = 0\)
4. Let \(\mathbf{u} = \langle 2, 0, -3 \rangle\) and \(\mathbf{v} = \langle 5, -3, 0 \rangle\).
   (a) Find \(3\mathbf{u} - 2\mathbf{v}\).
   (b) Find \(\mathbf{u} \cdot \mathbf{v}\).
   (c) Find a vector orthogonal to both \(\mathbf{u}\) and \(\mathbf{v}\).
   (d) Find a unit vector in the direction of \(\mathbf{u}\) and the direction cosines and angles of \(\mathbf{u}\).
   (e) Find the angle between the two vectors.
   (f) Find the projection of \(\mathbf{v}\) onto \(\mathbf{u}\).
5. Sketch the graphs of each of the following curves in space.
   (a) $2x + 3y + 6z = 12$
   (b) $x^2 + y^2 - z^2 = 1$
   (c) $x^2 + y^2 = 4$
   (d) $x = \frac{y^2}{4} + \frac{z^2}{9}$
   (e) $z = \ln x$  

6. Find the equation of the plane containing the points $(1, 1, 1)$, $(2, -3, 0)$, and $(0, 0, 1)$. Sketch the graph of this plane.

7. Sketch the graph of the plane with equation $5x + 3y + z = 15$.

8. Find parametric and symmetric equations for the line through the points $(1, 1, 2)$ and $(3, 6, 8)$.

9. Let $\mathbf{a} = (1, 2)$ and $\mathbf{b} = (3, 1)$. Find:
   (a) the projection of $\mathbf{a}$ onto $\mathbf{b}$.
   (b) the projection of $\mathbf{b}$ onto $\mathbf{a}$.
   (c) a graph to illustrate your answers to the two questions above.

10. A block of ice is dragged 20 feet across the floor using a force of 50 pounds. How much work is done if the force is applied at an angle of $30^\circ$ with respect to the horizontal?

11. Sketch the curve, including orientation, represented by the vector-valued function $\mathbf{r}(t) = 2 \cos t \mathbf{i} + 3 \sin t \mathbf{j} + \mathbf{k}$.

12. Evaluate the vector-valued function $\mathbf{r}(t) = t^2 \mathbf{i} + 3t \mathbf{j} + \frac{1 - \cos t}{t} \mathbf{k}$ at $t = \pi$, $t = \frac{\pi}{2}$, and find the limit $\lim_{t \to 0} \mathbf{r}(t)$.

13. Let $\mathbf{r}(t) = t \mathbf{i} + 3t \mathbf{j} + t^2 \mathbf{k}$ and $\mathbf{s}(t) = 4t \mathbf{i} + t^2 \mathbf{j} + t^4 \mathbf{k}$. Use the properties of the derivative to find the following.
   (a) $\mathbf{r}'(t)$
   (b) $D_t[3\mathbf{r}(t) - \mathbf{s}(t)]$
   (c) $D_t[\mathbf{r}(t) \cdot \mathbf{s}(t)]$
   (d) $D_t[\mathbf{r}(t) \times \mathbf{s}(t)]$
   (e) $D_t[\|\mathbf{r}(t)\|]$ for $t > 0$.

14. Evaluate the integrals:
   (a) $\int (e^t \mathbf{i} + \sin t \mathbf{j} + \cos t \mathbf{k}) \, dt$
   (b) $\int_0^1 (8t \mathbf{i} + t \mathbf{j} - \mathbf{k}) \, dt$

21. Find a vector-valued function $\mathbf{r}(t)$ such that $\mathbf{r}'(t) = 3t^2 \mathbf{j} + 6\sqrt{t} \mathbf{k}$ and $\mathbf{r}(0) = \mathbf{i} + 2\mathbf{j}$. 