Exam III Review Sheet
MATH 283, Winter 2005

This exam will cover sections 13.8-14.6 in your text. You should know general terms and definitions from each of these sections, review the homework given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Be able to change variables using Jacobians to make regions or integrands simpler.
2. Evaluating and sketching vector fields.
3. Identifying conservative vector fields in two dimensions.
4. Identifying conservative vector fields in three dimensions using the curl of the vector field.
5. Finding the potential function of a conservative two- or three-dimensional vector field.
6. Computing the curl and divergence of a vector field.
7. Finding piecewise smooth parameterizations of given curves.
8. Computing line integrals of a real-valued function along piecewise smooth curves.
9. Computing the line integral of a vector field on a smooth curve.
10. Computing line integrals in differential form.
11. Determining if a line integral is independent of path.
12. Computing the line integral along a closed curve through a conservative vector field.
13. Be able to state Green’s Theorem carefully and completely.
14. Using Green’s Theorem to compute line integrals around a simply connected region as double integrals.
15. Using Green’s Theorem to compute the area of a simply connected region as a line integral.
17. Finding normal vectors and tangent planes to parametric surfaces at a given point on the surface.
18. Computing the surface area of a parametric surface.
19. Computing surface integrals of real-valued functions over surfaces (both standard and parametric).
20. Computing the surface integral of a vector-field (Flux Integral) across both standard and parametric surfaces.

Below is a sampling of problems representative of the types you might see on the exam.

1. Determine whether or not the vector field \( \mathbf{F}(x, y) = (3x^2y^2 + 3y)i + (2x^3y + 3x)j \) is conservative. If it is, find a potential function for the vector field.
2. Let \( \mathbf{F}(x, y, z) = (\sin xy)i + zj + zyk \). Find curl \( \mathbf{F} \) and div \( \mathbf{F} \).
3. Sketch several representatives of the vector field \( \mathbf{F}(x, y) = -xi + yj \).
4. Evaluate the line integral \( \int_C x \, ds \) where \( C \) is the graph of \( y = x^2 - 1 \) joining the points \((-1, 0)\) and \((2, 3)\).
5. Let \( \mathbf{F} = (e^y + 2xy)i + (xe^y + x^2 + 1)j \). Let \( C \) be a smooth path connecting the points \((0, 0)\) and \((1, -1)\). Show that \( \int_C \mathbf{F} \cdot d\mathbf{R} \) is independent of the path \( C \) and find the value of this integral.
6. Use Green’s Theorem to evaluate \( \int_C (x - y) \, dx + 2x^2y \, dy \) where \( C \) is the triangle with vertices \((0, 0)\), \((0, 2)\), and \((1, 1)\).
7. Let \( R \) denote the region bounded by the parallelogram with vertices \((0, 0), (4, 0), (2, 4), \) and \((6, 4)\). Use the transformation \( x = u + v \) and \( y = 2u \) to evaluate the integral \( \int_{R} \int y(x - y) \, dx \, dy \).

8. Consider the integral 
\[
\int_{R} \int \frac{\sin(x - y)}{\cos(x + y)} \, dA
\]
where \( R \) is the triangular region bounded by the lines \( y = 0, \) \( y = x, \) and \( y + x = \frac{\pi}{2} \). Simplify the integrand by using the transformation \( u = x - y \) and \( v = x + y \). Write out (but do not evaluate) the new integral showing the limits of integration. Is the new integral simpler? If so, why?

9. Evaluate the line integral \( \int_{C} 2xy \, ds \) where \( C \) is the line beginning at \((2, 5)\) and ending at \((-2, 3)\).

10. Find the rectangular equation for the surface defined by \( r(u, v) = (\cos y)i + vj + (\sin u)k \).

11. Graph and find the area of the portion of the paraboloid \( r(u, v) = (u \cos v)i + (u \sin v)j + u^2k \) for \( 0 \leq u \leq 2 \) and \( 0 \leq v \leq 2\pi \).

12. Evaluate the line integral \( \int_{C} xy \, ds \) where \( C \) is the line joining \((1, 0)\) and \((3, 6)\).

13. Let \( F(x, y, z) = (3x^2y + ye^{xy})i + (x^2 + xe^{xy} + 1)j + k \), and let \( C \) be a smooth path connecting the points \((0, 0, 1)\) and \((1, 1, 3)\).
   (a) Show that \( F \) is a conservative vector field.
   (b) What does part (a) tell you about the integral \( \int_{C} F \cdot dr \)?
   (c) Find the value of the integral \( \int_{C} F \cdot dr \).

14. Evaluate \( \int_{S} \int \frac{xz}{y} \, dS \) if \( S \) is the portion of the cylinder \( x = y^2 \) that lies in the first octant that lies between the planes \( z = 0, \) \( z = 5, \) \( y = 1, \) and \( y = 4 \).

15. Let \( R \) be the rectangular region in the \( xy \)-plane bounded by the lines \( x = 0, \) \( x = 3, \) \( y = 0, \) and \( y = 2. \) Also, let \( f(x, y) = y + \frac{3}{4}x^2. \) Find the surface area \( S \) of the portion of the graph of \( f \) that lies above \( R \).

16. Let \( Q \) be the rectangular parallelepiped bounded by the coordinate planes and the graphs of \( x = 1, \) \( y = 1, \) and \( z = 3, \) and let \( S \) be the surface of \( Q. \) If \( F(x, y, z) = 2x^2z \hat{i} + xy^2 \hat{j} + x^3 \hat{k} \), find the flux of \( F \) over the surface \( S \).