Exam II Review Sheet
MATH 283, Winter 2008

This exam will cover sections 13.10-14.7 in your text. You should know general terms and definitions from each of these sections, review the homework given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Use Lagrange Multipliers and the method of Lagrange to minimize or maximize a function subject to given constraints.
2. Evaluate iterated integrals and set-up iterated integrals to find the area of a plane region.
3. Switch the order of integration in an iterated integral to ease the integration process.
4. State the definition of the double integral of a function \( f(x, y) \) over a plane region \( R \).
5. State Fubini’s Theorem, part 1.
6. Evaluate double integrals as iterated integrals and set-up double integrals to find the volume of a solid.
7. Change of variables in double integrals to polar form.
8. Using double integrals to find mass, moments, and centers of mass for planar lamina of variable density.
10. Using double integrals to find the area of a surface \( S \) given by \( z = f(x, y) \).
11. Set-up and evaluate triple integrals to find volume, including changing the order of integration.
12. Use triple integrals to find moments of mass and the center of mass for a solid of varying density.
13. Using triple integrals to find moments of inertia for solids of variable density.
14. Set-up or change variables in triple integrals to cylindrical or spherical coordinates.

Below is a sampling of problems representative of the types you might see on the exam.

1. Evaluate the integral \( \int_0^9 \int_{\sqrt{y}}^3 \sin x^3 \, dx \, dy \). Hint: reverse the order of integration.
2. Let \( f(x, y) = x^2y \). Find the maximum value of \( f(x, y) \) subject to: (a) \( x^2 + y^2 = 3 \) and (b) \( x^2 + y^2 \leq 3 \).
3. A solid is bounded by the cylinder \( r = a \) and sphere \( r^2 + z^2 = 4a^2 \) and the \( xy \)-plane. The density at a point \( P \) is directly proportional to the distance from \( P \) to the \( xy \)-plane. Find the mass of this solid.
4. Evaluate the integral \( \iiint_Q z(x^2 + y^2)^{\frac{1}{2}} \, dx \, dy \, dz \) where \( Q \) is the solid bounded above by the plane with equation \( z = 2 \) and below by the surface \( 2z = z^2 + y^2 \).
5. Use a triple integral to find the volume of the solid region above the plane with equation \( z = 1 \) and inside the sphere with equation \( x^2 + y^2 + z^2 = 2 \).
6. Find the volume and centroid of the solid bounded by the graphs of \( z = x^2 + y^2, x^2 + y^2 = 4 \), and \( z = 0 \).
7. Find the volume of the region in the spherical solid \( \rho \leq 4 \) after the solid cone \( \phi < \frac{\pi}{6} \) has been removed.
8. Define the double integral over a rectangular region. Then carefully explain the difference between the double integral and an iterated integral. What theorem allows one to write one type of integral in terms of the other?
9. Use a triple integral to find the volume of the solid region above the plane with equation \( z = 1 \) and inside the sphere with equation \( x^2 + y^2 + z^2 = 2 \).
10. Set-up (but do not solve) a triple integral giving the mass of the region bounded by the graphs of $x^2 + y^2 - z^2 = 0$ and $x^2 + y^2 = 4$ if the density function is $\rho(x, y, z) = z^2$.

11. Evaluate the triple integral $\int_0^1 \int_0^4 \int_0^{\sqrt{r^2-9x^2}} z \, dz \, dx \, dy$.

12. Evaluate the double integral $\int_0^\pi \int_0^{\sqrt{r}} x \sin y \, dx \, dy$.

13. A solid is bounded by the cylinder $r = a$, and a sphere $r^2 + z^2 = 4a^2$ and the $xy$-plane. The density at a point $P$ is directly proportional to the distance from $P$ to the $xy$-plane. Find the mass of this solid.

14. Find the centroid of a region bounded by a half-circle of radius $a$. Give reasons for any shortcuts that you might choose to use.

15. Evaluate $\int_0^1 \int_0^2 e^{y^2} \, dy \, dx$.

16. Use a triple integral to show that the volume of a sphere with radius $a$ is $\frac{4}{3} \pi a^3$.

17. Use cylindrical coordinates to find the volume in the first octant inside the cylinder $x^2 + y^2 = 2$ and below the plane with equation $x + y + z = 4$.

18. Find the surface area of the portion of the paraboloid $z = z^2 + y^2$ that lies below the plane $z = 1$.

19. Find the $x$-coordinate of the center of mass of the cube:

$$V = \{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1\}$$

if the density is proportional to the square of the distance from the origin.