Exam III Review Sheet  
MATH 312 Spring 2003

This exam will cover sections 6.1, 6.2, and 7.1-7.6 in your text. You should know general terms, definitions, and theorems from each of these sections, review the homework given for these sections, and pay particular attention to the subjects and practice problems mentioned below.

1. Working with power series (i.e. finding interval/radius of convergence, adding, multiplying, differentiating, etc.)
2. Existence theorem for power series solutions about an ordinary point and finding power series solutions about an ordinary point.
3. Identifying regular and irregular singular points of a D.E. and finding power series solutions about a regular singular point using Frobenius' Theorem.
4. Computing Laplace Transforms from the definition, transforms of basic functions, and sufficient conditions for the existence of a Laplace transform on a function $f(t)$.
5. Working with partial fractions to help compute inverse Laplace transforms.
6. Finding inverse Laplace transforms and using them to solve IVPs with constant coefficients.
7. Utilizing $t$-axis and $s$-axis translations to assist in computing Laplace transforms and solving IVPs.
8. Working with the Laplace transform of convolutions, integral equations, and periodic functions.

Below is a list of sample problems. This list is not all-inclusive, but does represent the types of problems you will see on the exam.

1. Does the existence theorem for series solutions guarantee power-series solutions for the IVPs given below? Why or why not? If yes, give a lower bound on the radius of convergence for these solutions.
   (a) $xy'' + y = 0$ subject to $y(0) = 1$, $y'(0) = 0$
   (b) $xy'' + y = 0$ subject to $y(1) = 1$, $y'(1) = 0$
   (c) $(x^2 - 2)y'' + 3y' + xy = e^x$ subject to $y(0) = 1$, $y'(0) = 6$

2. Find the first three terms of both series solutions to the following differential equations.
   (a) $2xy'' + 2y = 0$
   (b) $y'' - 4xy' - 4y = e^x$
   (c) $y' + x^3y = 0$

3. Find a power-series solution to the IVPs given below.
   (a) $y'' + y = 0$ subject to $y(0) = 0$, $y'(0) = 1$
   (b) $(x + 1)y'' - (2 - x)y' + y = 0$ subject to $y(1) = 2$, $y'(1) = -1$

4. Use the definition to find the Laplace transform of each of the following functions.
   (a) $f(t) = 1 - \cos kt$
   (b) $3t^2 + 4t - 1$
   (c) $e^{2t}$

5. Is the function $f(t) = 4t + e^t$ of exponential order? Prove your answer. Give an example of a (different?) function that is not of exponential order.
6. Write each of the functions below in terms of the Heaviside function $U(t)$. Then find the Laplace transform of each function.

(a) $f(t) = \begin{cases} 2 & \text{if } 0 \leq t < 3 \\ t - 1 & \text{if } 3 \leq t \end{cases}$

(b) $f(t) = \begin{cases} 0 & \text{if } 0 \leq t < \pi \\ \sin t & \text{if } \pi \leq t \end{cases}$

7. Find the Laplace transform of each of the following functions using any correct method.

(a) $f(t) = t\sin t$

(b) $f(t) = te^t$

(c) $f(t) = e^{3t}U(t - 4)$

(d) $f(t) = \begin{cases} 1 & \text{if } 0 \leq t < 1 \\ -1 & \text{if } 1 \leq t < 2 \end{cases}$ where $f$ is periodic with period 2.

8. Find the convolution of $\sin t$ and $t$.

9. Find each of the following inverse Laplace transforms.

(a) $\mathcal{L}^{-1}\left\{ \frac{1}{s + 2} \right\}$

(b) $\mathcal{L}^{-1}\left\{ \frac{1}{s^2 - 5s + 6} \right\}$

(c) $\mathcal{L}^{-1}\left\{ \frac{e^{-2s}}{s^2 + 9} \right\}$

10. Use a convolution integral to find the following inverse Laplace transform.

$\mathcal{L}^{-1}\left\{ \frac{a e^{-t}}{s^2 + a^2} \right\}$

11. Solve the following initial value problems using Laplace transforms.

(a) $y'' - 4y' + 4y = 0$ subject to $y(0) = 1$, $y'(0) = 0$

(b) $y'' - 4y' + 4y = t^3 e^{2y}$ subject to $y(0) = 0$, $y'(0) = 0$

(c) $y'' + y = f(t)$ subject to $y(0) = 1$, $y'(0) = 1$ where $f(t) = \begin{cases} 1 & \text{if } 1 \leq t < \frac{\pi}{2} \\ 0 & \text{if } \frac{\pi}{2} \leq t \end{cases}$

12. Use Laplace transforms to solve the integral equation below.

$f(t) + 2 \int_0^t f(\tau) \cos(t - \tau) d\tau = 4e^{-t} + \sin t$

13. Solve the system of differential equations given below using Laplace transforms.

\[ \begin{align*}
\frac{dx}{dt} + 3x + \frac{dy}{dt} &= 1 \\
\frac{dx}{dt} - x + \frac{dy}{dt} - y &= e^t
\end{align*} \]

Subject to $x(0) = 0$ and $y(0) = 0$