Solution Curves without a Solution
Section 2.1

- Direction Fields
- Initial Value Problems in Direction Fields
- Using Maple to Graph Direction Fields
- Autonomous First-Order ODEs
- Critical Points and Phase Portraits
- Attractors and Repellers
- Phase Portraits and Direction Fields
Direction Fields

A Direction Field for a first order differential equation is a graph in which each point is assigned a value equal to the slope that a solution curve would have if one passed through that point. These lines are called lineal elements.

\[
\frac{dy}{dx} = f(x, y)
\]

To determine the slope of lineal element at \((x,y)\), evaluate \(f(x,y)\)

Example: \[
\frac{dy}{dx} = 1 - xy
\]
Resulting Direction Field
Solve the following initial value problem using a direction field to graph the solution curve.

\[
\frac{dy}{dx} = \sin(y) \cos(y)
\]

Subject to:

a) \( y(0) = 1 \)  

b) \( y(1) = 0 \)
IVP Direction Field
IVP Solution, part a

a) $y(0) = 1$
IVP Solution, part b

\[ y(1) = 0 \]
Using Maple

Using maple, drawing direction fields and solutions becomes much easier:

```
with(plots): with(DEtools):
de:=diff(y(x),x) = x*exp(y(x));
initval:=[y(1)=2.5];
Deplot(de,y(x),x=-5..5,initval,y=-5..5);
```

See Maple Example
Autonomous DEs

Definition
A differential equation is called autonomous if the independent does not appear explicitly.

First Order Autonomous:

\[
\frac{dy}{dx} = f(y)
\]

\[
\frac{dy}{dx} = 2y - 4
\]

\[xy' = y\]
Critical Points

Definition
A **critical point** of an autonomous differential equation is a zero of the function \( f(y) \). If \( c \) is a critical point, then \( y(x) = c \) is a solution to the differential equation. It is called an **equilibrium solution**.

\[
\frac{dy}{dx} = y^2 - y^3
\]

\( y = 0, 1 \)

This breaks the plane into regions:
- dy/dx is negative
- dy/dx is positive
- semi-stable

**Attractor - asymptotically stable**
Phase Diagrams and DFs

\[ \frac{dy}{dx} = y(y-1)(y+1) \]

Repeller: unstable
Attractor: stable
Repeller: unstable