Linear Models
Section 3.1

- Population Growth/Decay
- Newton's Law of Cooling
- Kepler's Second Law of Planetary Motion
- Mixtures
- Circuit Analysis
Population Growth/Decay

The population of an endangered species is declining. In 1998, the population in a certain region was measured as 1230 individuals. In 2000, the population in that same region was measured as only 1080 individuals. Find a function $P(t)$ for the population size at any given time $t$ years after 1998. If this trend continues, approximately what should the population be today?

Starting Point: $\frac{dP}{dt} = kP$  \quad \text{Growth if } k > 0, \text{ decay if } k < 0$

Solution: $P(t) = 1230 e^{-0.065t}$

$P(2005 - 1998) = P(7) = 1230 e^{-0.455} \approx 780$
Newton's Law of Cooling

A thermometer is taken from an inside room to the outside, where the air temperature is five degrees Fahrenheit. After one minute, the thermometer reads fifty-five degrees Fahrenheit. After five minutes, the thermometer reads thirty degrees Fahrenheit. What was the temperature of the inside room?

Starting Point: \[ \frac{dT}{dt} = k(T - T_m) \]

What about \( k \)?

Solution: \[ T(t) = 5 + 59.46 \ e^{-0.173 \ t} \]
\[ T(0) = 5 + 59.46 = 64.46 \]
Kepler's Law of Planetary Motion

The angular momentum of a moving body of mass $m$ is given, in polar coordinates, by the expression:

$$L = mr^2 \frac{d\theta}{dt}$$

Assume that $L$ is constant and prove Kepler's Second Law. Namely, show that the radius vector joining the orbital focus to the body of mass sweeps out equal areas in equal time intervals.

$$A = \frac{L(b-a)}{2m}$$
Mixtures

A large tank is partially filled with 100 gallons of fluid in which 10 pounds of salt is dissolved. Brine, containing ½ pound of salt per gallon, is pumped into the tank at a rate of 6 gallons a minute. The well-mixed solution is then pumped out at a rate of 4 gallons per minute. Find the number of pounds of salt in the tank after 30 minutes.

Starting Point: \[
\frac{dA}{dt} = added - removed
\]

Solution: \[
A(t) = (50 + t) - \frac{100,000}{(50 + t)^2}
\]
\[
A(30) \approx 64.375
\]
Circuit Analysis

A 200-volt electromotive force is applied to the RC series circuit shown. Find the charge $q(t)$ on the capacitor if $i(0) = 0.3$. Determine the charge and current after .005 seconds, and determine the long term charge of the capacitor.

Resistor: $iR$

Capacitor: $\frac{1}{C} q$

Charge vs. Current: $i = \frac{dq}{dt}$

$$q(t) = \frac{1}{1000} - \frac{e^{-200t}}{500}$$

$q(0.005) \approx 0.1472$