MATH 312
Section 1.1: Definitions and Terminology

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Walla Walla College

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Outline

1. Syllabus
2. Introduction and Examples
3. Definitions and Terminology
4. Conclusion
Instructor and Resources

Instructor Information

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Resources

Text: A First Course in Differential Equations
Dennis G. Zill, 2005
Webpage: http://math.wwc.edu/courses/312/
Homework Expectations

Homework Procedures

Please keep the following homework procedures in mind.

- Daily 10-point assignments will be given.
- Assignments are due by 5:00 p.m. on the next class day.
- Assignments more than one day late will not be accepted.
- Your two lowest scores will be dropped.
- Follow the formatting instructions mentioned in the syllabus to receive full credit.
- Show all of your work to receive full credit.
- Present your solutions in an easy to follow and understand format.
- Work as a group, but turn in your own work only.
Exam Schedule

There will be three in class exams during the quarter.

- Exam I – 13 April, Chapters 1-3
- Exam II – 4 May, Chapters 4 and 5
- Exam III – 22 May, Chapters 6 and 7
- Final Exam – 5 June, Comprehensive

Exam Pointers

- In class exams may be moved with at least one week’s advance notice.
- The final exam may only be taken out of schedule after consultation with the Associate Academic Dean.
- As with homework, your solutions will be graded both on correctness and quality of presentation.
Assignment of Grades

Grade Weights

Your quarter average is based on the following categories and weights.

- Homework – 16%
- In Class Exams – $3 \times 18\% = 54\%$
- Final Exam – 30%

Grade Pointers

- Letter grades are assigned according to the table in your syllabus.
- Grades may be adjusted upward, but never downward.
- Read the college academic integrity statement!
What are Differential Equations?

What are differential equations? Let’s start by asking what an equation is.

**Example**

In algebra, you were asked to solve equations like $P(x) = 0$.

1. What is a solution to this equation?
2. How do we classify these equations?

**Definition 1.1**

An equation containing the derivatives of one or more dependent variables, with respect to one or more independent variables, is said to be a *differential equation* (DE).
Example 1: Recalling the Slope

One of the most basic ways we can view differential equations is as statements about slopes.

Example

Suppose that the slope of a certain graph \( y = f(t) \) is given by the equation \( m(t) \). Find a differential equation to model this situation.

Since \( \text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{dy}{dt} \) the equation is:

\[
\frac{dy}{dt} = m(t)
\]
Example 2: Filling a Balloon

The idea of change is central to derivatives, and so it plays an important part in differential equations.

Example

Let $y$ represent the radius of a balloon at time $t$. If this radius is increasing at the constant rate of 2 mm/sec, give a differential equation for the situation.

$$\frac{dy}{dt} = 2$$
Example 3: A Growing Population

Our final introductory example looks at population growth and differential equations.

Example

Let $P$ represent a population at time $t$, and suppose that $P$ is increasing at a rate proportional to $P$ itself. Find a differential equation for $P$.

\[
\frac{dP}{dt} = \lambda P
\]
Classification of DEs

Just as with polynomial equations, differential equations can be classified based on the form of the equation.

**Classification by Type**

Differential equations can be classified into two distinct types:

- **Ordinary Differential Equations**
  are equations which have a single independent variable.

- **Partial Differential Equations**
  are equations with multiple independent variables.

**Classification by Order**

Differential equations can also be classified by the order of the equation. The order of a DE is the highest order derivative appearing in the equation.
Linear vs. Nonlinear DEs

Another important way to classify differential equations is as either linear or non-linear.

**Linear Differential Equations**

Below is the general $n$th order linear ODE.

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = g(x)$$

Notice the following properties, which make this linear.

- All coefficients depend only on $x$.
- The function $y$ and its derivatives appear to at most the first power.
- The function $g(x)$ depends only on $x$. 
Classification Exercises

Now use the classifications above to classify the following differential equations.

Example

\[ 3 \sin(x)y - xy' = 3^x \]

Example

\[ 2x \left( \frac{dy}{dx} \right)^2 + 3y = x \]

Example

\[ (1 - y)y_{xx} + \sin y_{zx} = 0 \]
Various DE Notations

In the last slide, we saw several examples using different notation for derivatives. Since we are focused on differential equations, it is important to know these various representations.

Representations of a Derivative

Leibniz Notation: $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \ldots, \frac{d^ny}{dx^n}$

Prime Notation: $y', y'', \ldots, y^{(n)}$

Newton’s Dot Notation: $\dot{y}, \ddot{y}, \ldots$

Subscript Notation: $y_x, y_z, y_{xx}, \ldots$
Explicit Solutions

Now that we know more about what DEs are, it is time to ask about their solutions.

**Definition 1.2**

Any function \( \varphi \) defined on an interval \( I \) and possessing at least \( n \) continuous derivatives on \( I \), which when substituted into an \( n \)th order differential equation reduces the equation to an identity, is said to be a *solution* of the equation on the interval.

**Example**

Verify that the function \( y = e^{-x^2} \int_0^x e^{t^2} \, dt + e^{-x^2} \) is a solution to the ODE \( y' + 2xy = 1 \).

**Particular Solution**

In fact, this is a *particular solution* from an infinite family of solutions.
Implicit Solutions

You are familiar with explicitly and implicitly defined functions. The same phenomena appears in solutions to DEs.

**Definition 1.3**

A relation $G(x, y) = 0$ is said to be an *implicit solution* of an ODE $F(x, \varphi(x), \varphi'(x), \ldots, \varphi^{(n)}(x)) = 0$ on an interval $I$, provided there exists at least one function $\varphi$ that satisfies the relation as well as the differential equation on $I$.

**Example**

Show that $G(x, y) = x - 3y^2 + 4 = 0$ is an implicit solution to the ODE $y' = \frac{1}{6y}$.

**Example**

Can you find two explicit solutions to this DE?
Important Concepts

Things to Remember from Section 1.1

1. Classification of Differential Equations by:
   - Type
   - Order
   - Linearity

2. Differential Equation Notation

3. Particular Solutions vs. Families of Solutions

4. Explicit vs. Implicit Solutions