MATH 461, Exam II
Fall 2006 Oral Examination Instructions

Please select one of the following theorems to present and prove during your oral exam. You will need to make your presentation without notes and may be asked a few questions about the proof after your presentation. Your grade will be based on mathematical accuracy, your ability to answer questions, and presentation style.

1. Cyclic Subgroup Orders Theorem
   Let \( G = \langle g \rangle \) be a cyclic group of order \( n \). Then for any \( k \in \mathbb{Z}^+ \), \( \langle g^k \rangle = \langle g^{(n,k)} \rangle \) and \( |g^k| = \frac{n}{(n,k)} \).

2. Cayley’s Theorem
   Every group is isomorphic to a group of permutations.

3. Permutation Order Theorem
   Let \( \sigma \) be a permutation in \( S_n \). The order of \( \sigma \) in the group \( S_n \) is the least common multiple of the lengths of the disjoint cycles which make up \( \sigma \).

4. Lagrange’s Theorem
   If \( G \) is a finite group and \( H \) is a subgroup of \( G \), then \( |H| \) divides \( |G| \) and the number of distinct left cosets of \( H \) in \( G \), called the index of \( G \) in \( G \), is \( [G:H] = \frac{|G|}{|H|} \).
   (Note: You will need to prove that the left cosets of \( H \) form a partition of \( G \) as a part of this theorem.)

5. Direct Products and Cyclic Groups Theorem
   Let \( G \) and \( H \) be finite cyclic groups. Then the group \( G \times H \) is cyclic if and only if \( |G| \) and \( |H| \) are relatively prime.

6. Inner Automorphism Theorem
   The set of inner automorphisms of a group \( G \), \( \text{Inn}(G) = \{ \phi_g \mid g \in G \} \) is a normal subgroup of \( \text{Aut}(G) \), the group of automorphisms of \( G \).

Available Exam Times

Once you have selected a theorem, choose one of the time slots from the list below and email your instructor (duncjo@wwc.edu) to reserve that time. Exam times will be assigned on a first-come first-served basis.