Survey of Calculus: Rules for Differentiation

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Example

The ideal output for a certain factory worker is thirty items per hour. A new worker produces fewer items at first, and then produces more items as he or she learns. Draw the graph of the number of items per hour that a new worker can produce after a certain number of days on the job. Notice, that as time increases and as the worker learns, the number of items increases toward the ideal output.

1. How many items per hour can be produced by a worker after one day on the job?
2. How many items per hour can be produced after five days?
3. Is the rate of learning faster during the second day or during the seventh day? Use the word "derivative" in your answer.
4. What do you think the rate of leaning would be after, say, twenty years?

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- General power rule

$$\frac{d}{dx}([g(x)]^r) = r \cdot [g(x)]^{r-1} \cdot \frac{d}{dx}[g(x)].$$

Rate of Change

Recall two properties of the tangent line to the graph at the point $P = (a, f(a))$:

- The tangent line at P has slope $f'(a)$.
- The tangent line at P is the line that best approximates the graph near P.

Also recall that the rate of change of a linear function is equal to its slope. Since we can think of the graph of f near P as being a line with slope $f'(a)$, we can also think of $f'(a)$ as being the rate of change of $f(x)$ at $x = a$.

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- $$f(a+h) = f(a) + f'(a)h$$

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- Use Linear Approximation to explain how derivatives can be used to estimate marginal quantities.

Example – Problem 62,Page 117

A toy company introduces a new video game on the market. Let $S(x)$ denote the number of videos sold on day x, that is, x days after introduction. Let n denote a positive integer and interpret $S(n)$, $S'(n)$, and $S(n) + S'(n)$.